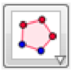
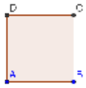
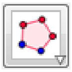
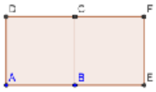
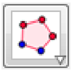
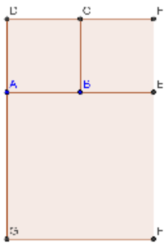
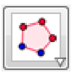
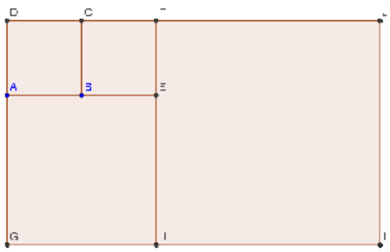
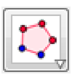
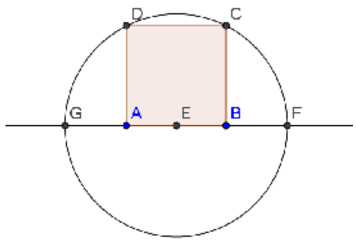
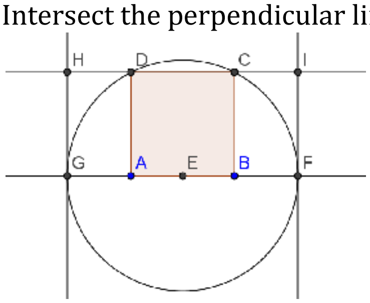
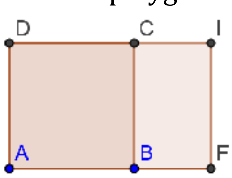
Name: ______________________________ Date: _______________

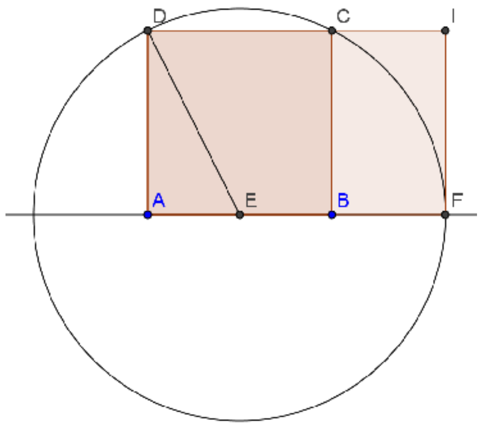
Investigating the Golden Ratio

	1. Use the **Regular Polygon** tool to construct square *ABCD*. 
	2. Using the **Regular Polygon** tool, click on vertex *C* then *B* to construct square *CBEF*. 
	3. Using the **Regular Polygon** tool, click on vertex *E* then *A* to construct square *EAGH*. 
	4. Using the **Regular Polygon** tool, click on vertex *F* then *H* to construct square *FHIJ*. 
	5. Continue in this manner (zoom out if necessary) for several more iterations. 

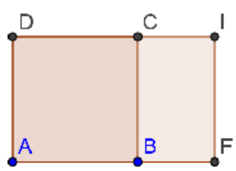
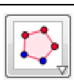
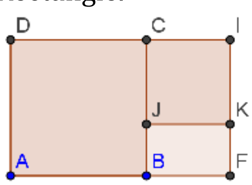
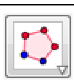
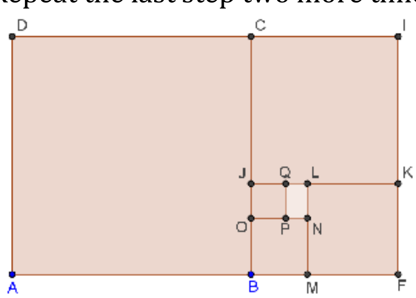
	6. Assume the square *ABCD* is a unit square. List the side lengths of each square constructed, in order, beginning with square *ABCD*. What pattern do you notice in this sequence of side lengths?
	7. Record the ratio of each side length to the side length of the preceding square, beginning with the first two squares, *CBEF* and *ABCD*. What do you notice about these ratios? $\varphi \approx 1.6180339887$
	8. A Golden Rectangle is a rectangle for which the ratio of the longer side to the shorter side is $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887$. Is rectangle *DOQR* a Golden Rectangle? Explain.

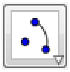
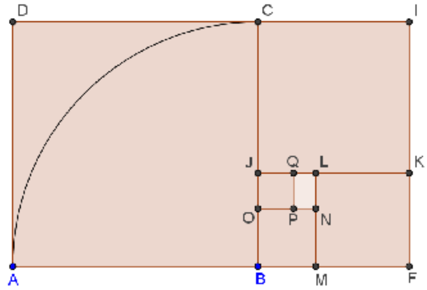
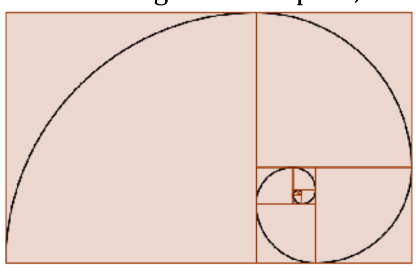
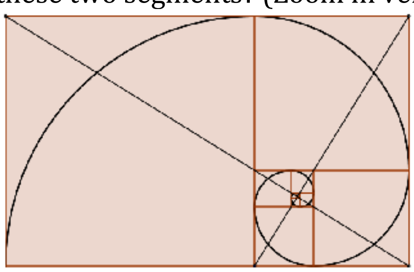
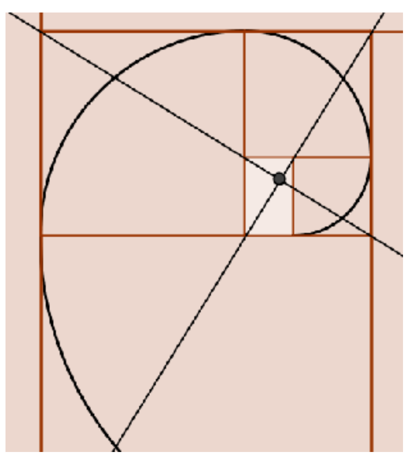
Constructing a Golden Rectangle

	1. Use the **Regular Polygon** tool to construct square *ABCD*.
	2. Construct a line passing through points *A* and *B*.
	3. Construct the midpoint of $\overline{AB}$ and label it point *E*.
	4. Construct a circle centered at *E* and passing through point *D*.
	5. Intersect the circle with $\overleftrightarrow{AB}$ and label the points of intersection *G* and *F*. 
	6. Construct a line passing through points *D* and *C*.
	7. Construct a line passing through point *G* that is perpendicular to $\overleftrightarrow{AB}$.
	8. Construct a line passing through point *H* that is perpendicular to $\overleftrightarrow{AB}$.
	9. Intersect the perpendicular lines with $\overleftrightarrow{CD}$. Label these points *H* and *I*. 
	10. Hide the circle and all four lines. Hide points *H, G,* and *E*.
	11. Construct polygon *ADIF*. *ADIF* is a Golden Rectangle. 
	12. Describe the ratio of *AF* to *DA*.

	13. Given square *ABCD*, let *E* be the midpoint of $\overline{AB}$ and the center of the circle with radius *ED*. Let $AD = x$. Show that $\frac{AF}{DA} = \varphi = \frac{1+\sqrt{5}}{2}$. (Hint: find *AE* and *ED*) 

Constructing a Golden Spiral

	1. Construct a Golden Rectangle *ADIF* (see previous activity). 
	2. Use the **Regular Polygon** tool to construct square *ICJK*. *BJKF* is also a Golden Rectangle. 
	3. Repeat the last step two more times, forming square *FKLM* and *BMNO*. 

	4. Construct an arc from C to A with radius AB. To do this, select the **Circular Arc with Center between Two Points** tool. Click point B, followed by points C and A. 
	5. Continue with the next smaller square, and so on, as far as you can get. This is called the Logarithmic Spiral, or the Golden Spiral. 
	6. Construct $\overline{DF}$ and $\overline{BI}$. What do you notice about the point of intersection of these two segments? (Zoom in very close to get a better view).   This point of intersection is nicknamed “The Eye of God.”

Constructing a Golden Triangle

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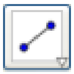
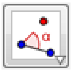
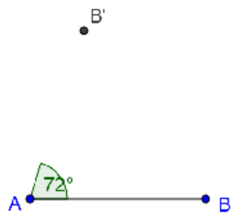
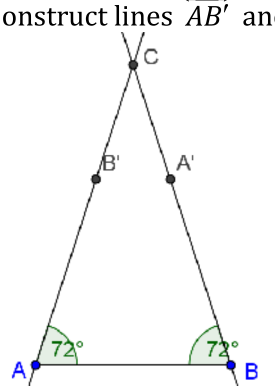
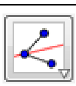
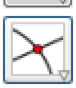
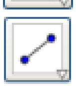
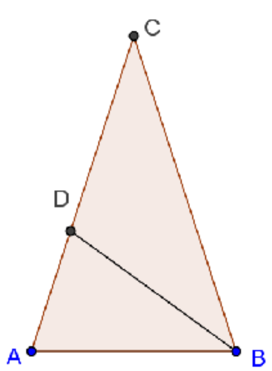
<tr><td></td><td>1. Construct $\overline{AB}$.</td></tr>
<tr><td></td><td>2. Use the Angle with Given Size tool to construct a base angle of 72°. To do this, select the Angle with Given Size tool, then click point B followed by point A. A box will pop up asking for the angle measure. Enter 72, select counter clockwise, and hit Enter.

</td></tr>
<tr><td></td><td>3. Use the Angle with Given Size tool to construct the other base angle of 72°. To do this, select the Angle with Given Size tool, then click point A followed by point B. A box will pop up asking for the angle measure. Enter 72, select clockwise, and hit Enter.</td></tr>
<tr><td></td><td>4. Construct lines $\overleftrightarrow{AB'}$ and $\overleftrightarrow{BA'}$. Construct the point of intersection C.

</td></tr>
<tr><td></td><td>5. Hide the two lines and the angles marked 72°. Hide points B' and A'. Use the polygon tool to construct isosceles triangle $\triangle ABC$.</td></tr>
<tr><td>

</td><td>6. Bisect $\angle B$ and construct point D, the intersection of $\overline{AC}$ and the bisector of $\angle B$. Then hide the line and construct $\overline{BD}$.

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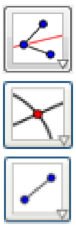
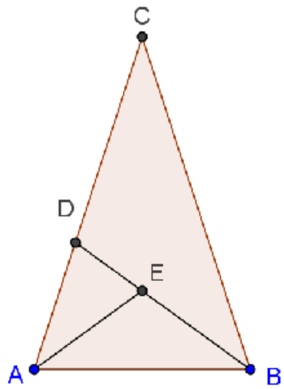
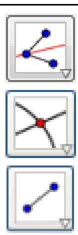
7. What is the relationship between $\triangle ABC$ and $\triangle DAB$? Justify your response.

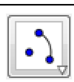


8. Bisect $\angle A$ and construct point E, the intersection of $\overline{BD}$ and the bisector of $\angle A$. Then hide the line and construct $\overline{AE}$.

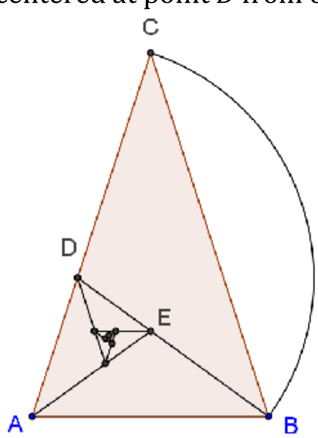


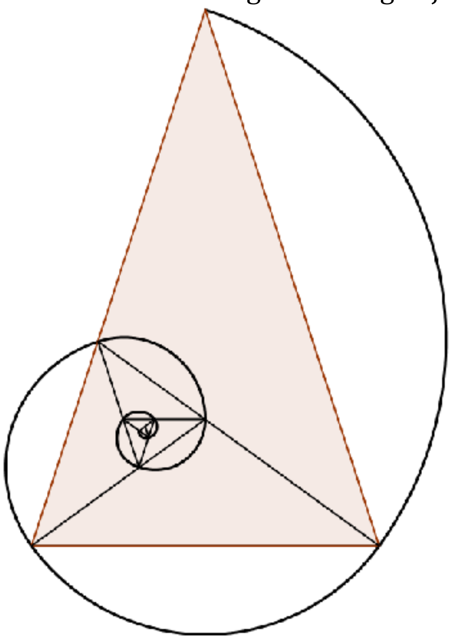


9. Continue bisecting the next angle in the sequence in a clockwise direction (first we bisected $\angle B$, then $\angle A$. Next we'll bisect $\angle D$, $\angle E$, $\angle F$, $\angle G$, $\angle H$, and so on. Hide the lines as you go, and replace them with segments.



10. Use the **Circular Arc with Center between Two Points** tool to construct an arc centered at point D from C to B.



	11. Continue constructing arcs along adjacent legs of subsequent triangles. 
	12. In each of the subsequently formed triangles similar to the original $\triangle ABC$, the ratio of the leg to the base is $\varphi = \frac{1+\sqrt{5}}{2}$.