1-4

1. Plan

Objectives
1 To solve and graph inequalities
2 To solve and write compound inequalities

Examples
1 Solving and Graphing Inequalities
2 No Solutions or All Real Numbers as Solutions
3 Real-World Connection
4 Compound Inequality Containing *And*
5 Compound Inequality Containing *Or*
6 Real-World Connection

Professional Development

Math Background

Solving inequalities is much like solving equations. An important difference is that multiplying or dividing both sides by a negative number reverses the direction of the inequality symbol.

The work with compound inequalities in this lesson prepares students for solving absolute value inequalities in the next lesson.

More Math Background: p. 2D

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:
Properties of Real Numbers
Lesson 1-1: Example 3
Extra Skills and Word Problems Practice, Ch. 1
Solving Equations
Lesson 1-3: Examples 1, 2
Extra Skills and Word Problems Practice, Ch. 1

Solving Inequalities

What You'll Learn
- To solve and graph inequalities
- To solve and write compound inequalities

. . . And Why
To analyze quality control, as in Example 6

Check Skills You'll Need GO for Help Lessons 1-1 and 1-3

State whether each inequality is true or false.
1. $5 < 12$ true **2.** $5 < -12$ false **3.** $5 \geq 12$ false
4. $5 \leq -12$ false **5.** $5 \leq 5$ true **6.** $5 \geq 5$ true

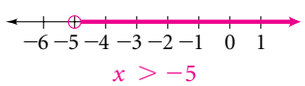
Solve each equation.
7. $3x + 3 = 2x - 3$ -6 **8.** $5x = 9(x - 8) + 12$ 15

New Vocabulary • compound inequality

1 Solving and Graphing Inequalities

As with an equation, the solutions of an inequality are the numbers that make it true.

An equation such as $-2x = 10$ has only one solution, -5. On the other hand, the inequality $-2x < 10$ is true for many values of x, such as -4.99, -1, and 100. The solutions of $-2x < 10$ are all the numbers x such that $x > -5$, as shown in the graph at the right.



$x > -5$

GO for Help
To review the properties of equality, go to p. 18.

The properties for solving inequalities are similar to the properties for solving equations. The exception occurs when you multiply or divide each side by a negative quantity. Notice that you can obtain $x > -5$ from $-2x < 10$ by dividing each side by -2 and *reversing* the inequality symbol.

The following properties are for $\leq$. There are similar properties for $<$, $>$, and $\geq$.

Key Concepts

Property	Properties of Inequalities
Let a, b, and c represent real numbers.	
Transitive Property	If $a \leq b$ and $b \leq c$, then $a \leq c$.
Addition Property	If $a \leq b$, then $a + c \leq b + c$.
Subtraction Property	If $a \leq b$, then $a - c \leq b - c$.
Multiplication Property	If $a \leq b$ and $c > 0$, then $ac \leq bc$. If $a \leq b$ and $c < 0$, then $ac \geq bc$. ← You must reverse the inequality symbol when c is negative.
Division Property	If $a \leq b$ and $c > 0$, then $\frac{a}{c} \leq \frac{b}{c}$. If $a \leq b$ and $c < 0$, then $\frac{a}{c} \geq \frac{b}{c}$. ← You must reverse the inequality symbol when c is negative.

Differentiated Instruction Solutions for All Learners

Special Needs L1
For Example 6, have students sketch a diagram to illustrate how to trim a 5-in. paper strip so that it is 3 in. long with a tolerance of $\frac{1}{4}$ in.
learning style: tactile

Below Level L2
If students are having difficulty understanding the concept of a solution to an inequality, use substitution to show that many different numbers satisfy the inequality.
learning style: visual

1 EXAMPLE Solving and Graphing Inequalities

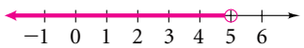
Solve each inequality. Graph the solution.

a. $3x - 12 < 3$

$3x - 12 < 3$

$3x < 15$ **Add 12 to each side.**

$x < 5$ **Divide each side by 3.**



Graph the solution.

Check First check the boundary point: $3(5) - 12 = 3$. ✓
Then check another point on the graph, such as 4: $3(4) - 12 < 3$. ✓

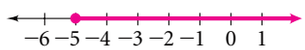
b. $6 + 5(2 - x) \leq 41$

$6 + 10 - 5x \leq 41$ **Distributive Property**

$16 - 5x \leq 41$ **Simplify.**

$-5x \leq 25$ **Subtract 16 from each side.**

$x \geq -5$ **Divide each side by −5 and reverse the inequality.**



Graph the solution.

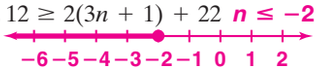
Check First check the boundary point: $6 + 5[2 - (-5)] = 41$. ✓
Then check another point, such as −4: $6 + 5[2 - (-4)] \leq 41$. ✓

Quick Check

1 Solve each inequality. Graph the solution.

a. $3x - 6 < 27$ ***x* < 11**

b. $12 \geq 2(3n + 1) + 22$ ***n* ≤ −2**



Some inequalities have no solution, and some are true for all real numbers.

2 EXAMPLE No Solutions or All Real Numbers as Solutions

Solve each inequality. Graph the solution.

a. $2x - 3 > 2(x - 5)$

$2x - 3 > 2x - 10$ **Distributive Property**

$2x > 2x - 7$ **Add 3 to each side.**

$0 > -7$ **Subtract 2*x* from each side.**

The last inequality is always true, so $2x - 3 > 2(x - 5)$ is always true. All real numbers are solutions.

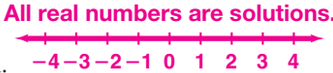
b. $7x + 6 < 7(x - 4)$

$7x + 6 < 7x - 28$ **Distributive Property**

$6 < -28$ **Subtract 7*x* from each side.**

The last inequality is always false, so $7x + 6 < 7(x - 4)$ is always false. It has no solution.

All real numbers are solutions.



Video Tutor Help
Visit: PHSchool.com
Web Code: age-0775

Quick Check

2 **a.** Solve $2x < 2(x + 1) + 3$. Graph the solution.

b. Solve $4(x - 3) + 7 \geq 4x + 1$. Graph the solution. **no solutions**

c. **Critical Thinking** If possible, find values of a such that $2x + a > 2x$ has no solution. Then find values of a such that all real numbers are solutions.
values of *a* less than or equal to 0; values of *a* greater than 0

2. Teach

Guided Instruction

1 EXAMPLE Teaching Tip

Remind students that when they graph an inequality, they should use an open circle at the boundary point if the boundary point is not a solution.

2 EXAMPLE Visual Learners

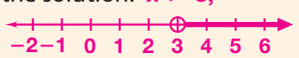
When an inequality has no solution, students may want to represent this situation graphically by drawing a number line without any shading.

3 EXAMPLE Math Tip

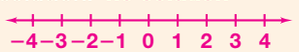
The inequality may also be written as $500 \leq 200 + 0.25x$. It is customary (but not necessary) to write the final inequality in the solution so that the variable is on the left side of the inequality symbol.

Additional Examples

1 Solve $-2x < 3(x - 5)$. Graph the solution. ***x* > 3;**



2 Solve $7x \geq 7(2 + x)$. Graph the solution. **no solution**



3 A real estate agent earns a salary of $2000 per month plus 4% of the sales. Find the sales if the salesperson is to have a monthly income of at least $5000. **greater than or equal to $75,000**

Advanced Learners L4
Have students try to create inequalities that have a single real-number solution, no solution, or the set of all real numbers as the solution.

learning style: verbal

English Language Learners ELL
Have volunteers take turns reading aloud the exposition for this lesson. Have students give real-life examples using *and* or *or*, e.g., "Which would you rather have: a $10 AND a $5 bill or a $10 OR a $5 bill?"

learning style: verbal

Guided Instruction

4 EXAMPLE Teaching Tip

You may wish to use a transparency and erasable color markers to show the graph of $x > -9$ in yellow and the graph of $x < 6$ in blue. The part of the number line that is green is the solution of the compound inequality.

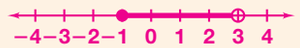
5 EXAMPLE Teaching Tip

Adapt the Teaching Tip for Example 4. This time the solution of the compound inequality is any part of the number line that has color.

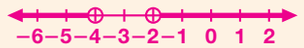
PowerPoint

Additional Examples

4 Graph the solution of $2x - 1 \leq 3x$ and $x > 4x - 9$.



5 Graph the solution of $3x + 9 < -3$ or $-2x + 1 < 5$.



6 A strip of wood is to be 17 cm long with a tolerance of ± 0.15 cm. How much should be trimmed from a strip 18 cm long to allow it to meet specifications?
at least 0.85 cm and no more than 1.15 cm

Resources

- Daily Notetaking Guide 1-4 L3
- Daily Notetaking Guide 1-4—Adapted Instruction L1

Closure

Ask: *What is one important difference between solving equations and solving inequalities?* **Answers may vary. Sample: If you multiply or divide both sides of an inequality by a negative number, you change the direction of the inequality symbol.**

pages 29–31 Exercises

1. 

3 EXAMPLE Real-World Connection

Revenue The band shown at the left agrees to play for \$200 plus 25% of the ticket sales. Find the ticket sales needed for the band to receive at least \$500.

Relate \$200 + 25% of ticket sales $\geq$ \$500

Define Let x = ticket sales (in dollars).

Write
$200 + 0.25x \geq 500$
$0.25x \geq 300$ **Subtract 200 from each side.**
$x \geq 1200$ **Divide each side by 0.25.**

The ticket sales must be greater than or equal to \$1200.

Quick Check

3 A salesperson earns a salary of \$700 per month plus 2% of the sales. What must the sales be if the salesperson is to have a monthly income of at least \$1800?
at least \$55,000

2 Compound Inequalities

A **compound inequality** is a pair of inequalities joined by *and* or *or*.

Examples:
- $-1 < x$ and $x \leq 3$, which you can also write as $-1 < x \leq 3$
- $x < -1$ or $x \geq 3$

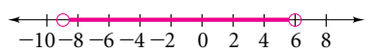
For: Inequalities Activity
Use: Interactive Textbook, 1-4

To solve a compound inequality containing *and*, find all values of the variable that make both inequalities true.

4 EXAMPLE Compound Inequality Containing *And*

Graph the solution of $3x - 1 > -28$ and $2x + 7 < 19$.

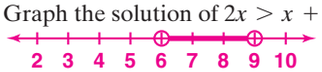
$3x - 1 > -28$ and $2x + 7 < 19$
$3x > -27$ | $2x < 12$
$x > -9$ and $x < 6$



This compound inequality can be rewritten as $-9 < x < 6$.

Quick Check

4 Graph the solution of $2x > x + 6$ and $x - 7 < 2$.

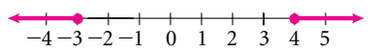


To solve a compound inequality containing *or*, find all values of the variable that make at least one of the inequalities true.

5 EXAMPLE Compound Inequality Containing *Or*

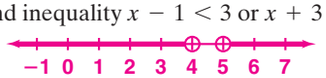
Graph the solution of $4y - 2 \geq 14$ or $3y - 4 \leq -13$.

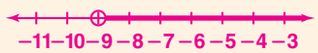
$4y - 2 \geq 14$ or $3y - 4 \leq -13$
$4y \geq 16$ | $3y \leq -9$
$y \geq 4$ or $y \leq -3$

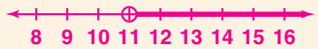


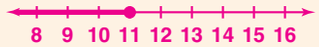
Quick Check

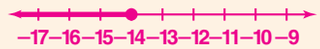
5 Solve the compound inequality $x - 1 < 3$ or $x + 3 > 8$. Graph the solution.
***x* < 4 or *x* > 5**



2. 

3. 

4. 

5. 

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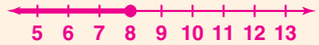
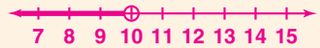
7. 

8. 

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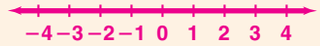
10. 

6 EXAMPLE Real-World Connection

Multiple Choice The ideal length of a bolt is 13.48 cm. The length can vary from the ideal by at most 0.03 cm. A machinist finds one bolt that is 13.67 cm long. By how much should the machinist decrease the length so the bolt can be used?

(A) between 13.45 cm and 13.51 cm (B) between 13.64 cm and 13.70 cm

(C) between 0.16 cm and 0.22 cm (D) between 0.13 cm and 0.19 cm

Relate minimum length ≤ final length ≤ maximum length

Define Let x = number of centimeters to remove.

Write
$$13.48 - 0.03 \le 13.67 - x \le 13.48 + 0.03$$
$$13.45 \le 13.67 - x \le 13.51 \quad \text{Simplify.}$$
$$-0.22 \le -x \le -0.16 \quad \text{Subtract 13.67.}$$
$$0.22 \ge x \ge 0.16 \quad \text{Multiply by } -1.$$

The machinist must remove at least 0.16 cm and no more than 0.22 cm. The answer is C.

Test-Taking Tip

When you multiply by a negative number across a compound inequality, remember to reverse both inequality symbols.

Quick Check 6 The plans for a circular plastic part in a medical instrument require the diameter to be within 0.2 in. of 1.5 in. A machinist finds that the diameter is now 1.73 in. By how much should the machinist decrease the diameter?

by at least 0.03 in., but by no more than 0.43 in.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (page 27)

GO for Help

Solve each inequality. Graph the solution. 1–10. See margin p. 28 for graphs. 11-13. See back of book for graphs.

1. $-12 \ge 24x$ $x \le -\frac{1}{2}$
2. $-7k < 63$ $k > -9$
3. $8a - 15 > 73$ $a > 11$
4. $57 - 4t \ge 13$ $t \le 11$
5. $-18 - 5y \ge 52$ $y \le -14$
6. $14 - 4y \ge 38$ $y \le -6$
7. $4(x + 3) \le 44$ $x \le 8$
8. $2(m - 3) + 7 < 21$ $m < 10$
9. $4(n - 2) - 6 > 18$ $n > 8$
10. $9(x + 2) > 9(x - 3)$ All real numbers are solutions.
11. $6x - 13 < 6(x - 2)$ All real numbers are solutions.
12. $-6(2x - 10) + 12x \le 180$ All real numbers are solutions.
13. $-7(3x - 7) + 21x \ge 50$ no solutions

Example 3 (page 28)

Solve each problem by writing an inequality.

14. The length of a picture frame is 3 in. greater than the width. The perimeter is less than 52 in. Describe the dimensions of the frame. The width is less than 11.5 in., and the length is 3 in. greater than the width.
15. The lengths of the sides of a triangle are in the ratio 5 : 6 : 7. Describe the length of the longest side if the perimeter is not more than 54 cm. The longest side is less than 21 cm.
16. Find the lesser of two consecutive integers with a sum greater than 16. The smaller number is an integer greater than or equal to 8.
17. The cost of a field trip is $220 plus $7 per student. If the school can spend at most $500, how many students can go on the field trip? 40 students

Example 4 (page 28)

Solve each compound inequality. Graph the solution. 18–21. See margin for graphs.

18. $2x > -10$ and $9x < 18$ $-5 < x < 2$
19. $3x \ge -12$ and $8x \le 16$ $-4 \le x \le 2$
20. $6x \ge -24$ and $9x < 54$ $-4 \le x < 6$
21. $7x > -35$ and $5x \le 30$ $-5 < x \le 6$

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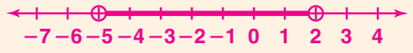

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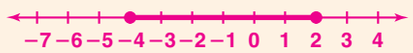

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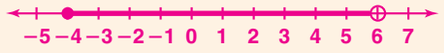

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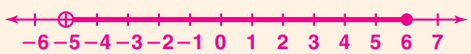


3. Practice

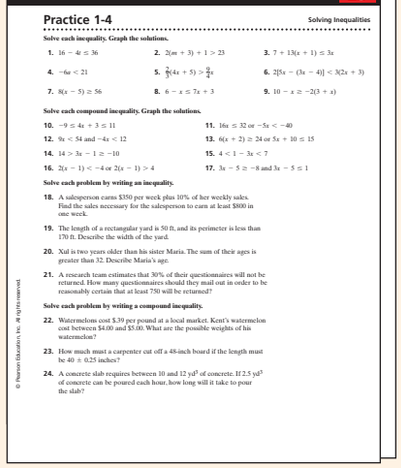
Assignment Guide

1	A B	1-17, 29-35, 38-40
2	A B	18-28, 36-37, 41-50
C	Challenge	51-55
	Test Prep	56-61
	Mixed Review	62-69

Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 17, 27, 35, 37, 38.

Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Practice L3


Practice 1-4 Solving Inequalities

Solve each inequality. Graph the solutions.

1. $16 - 4t \le 36$
2. $2(m + 3) + 1 > 23$
3. $7 + 13(x + 1) \le 3x$
4. $-6a < 21$
5. $\frac{2}{3}(4x + 5) > \frac{2}{4}x$
6. $2[5x - (3x - 4)] < 3(2x + 3)$
7. $8(x - 5) \ge 56$
8. $6 - x \le 7x + 3$
9. $10 - x \ge -2(3 + x)$

Solve each compound inequality. Graph the solutions.

10. $-9 \le 4x + 3 \le 11$
11. $16s \le 32$ or $-5s < -40$
12. $9x < 54$ and $-4x < 12$
13. $6(x + 2) \ge 24$ or $5x + 10 \le 15$
14. $14 > 3x - 1 \ge -10$
15. $4 < 1 - 3x < 7$
16. $2(x - 1) < -4$ or $2(x - 1) > 4$
17. $3x - 5 \ge -8$ and $3x - 5 \le 1$

Solve each problem by writing an inequality.

18. A salesperson earns $350 per week plus 10% of her weekly sales. Find the sales necessary for the salesperson to earn at least $800 in one week.
19. The length of a rectangular yard is 50 ft, and its perimeter is less than 170 ft. Describe the width of the yard.
20. Xul is two years older than his sister Maria. The sum of their ages is greater than 32. Describe Maria's age.
21. A research team estimates that 30% of their questionnaires will not be returned. How many questionnaires should they mail out in order to be reasonably certain that at least 750 will be returned?

Solve each problem by writing a compound inequality.

22. Watermelons cost $.39 per pound at a local market. Kent's watermelon cost between $4.00 and $5.00. What are the possible weights of his watermelon?
23. How much must a carpenter cut off a 48-inch board if the length must be 40 ± 0.25 inches?
24. A concrete slab requires between 10 and 12 yd^3 of concrete. If 2.5 yd^3 of concrete can be poured each hour, how long will it take to pour the slab?

4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Solve $-2(x - 3) \geq 4$. Graph the solution. $x \leq 1$

2. Solve $-5(4 - x) < 5x$. Graph the solution. all real numbers

3. Graph the solution of $3x + 4 \geq 1$ and $-2x + 7 \geq 5$.

4. A copper wire is to have a length of 16 cm with a tolerance of $\pm$ 0.02 cm. How much must be trimmed from a wire that is 18 cm long for it to meet specifications? at least 1.98 cm and no more than 2.02 cm

Alternative Assessment

Students work in pairs. They use masking tape to create a number line from −10 to 10 on a desk. Each student places his or her pencil above the number line, using the tip of the pencil as an arrowhead, indicating a single inequality such as $x \geq 3$. Pencils can point in the same or opposite direction and may or may not have an overlapping region. Each group switches stations and writes down two compound inequalities, one inequality assuming that the pencils represent a conjunction and the other assuming a disjunction. Students graph both inequalities and create problems that result in the solution sets represented by the graphs. Students discuss the results.

Example 5 (page 28)

Solve each compound inequality. Graph the solution. See margin for graphs.

22. $4x < 16$ or $12x > 144$ $x < 4$ or $x > 12$

23. $3x \geq 3$ or $9x < 54$ All real numbers are solutions.

24. $8x > -32$ or $-6x \leq 48$ $x \geq -8$

25. $9x \leq -27$ or $4x \geq 36$ $x \leq -3$ or $x \geq 9$

Example 6 (page 29)

Solve each problem by writing a compound inequality.

26. A baker needs between 40 lb and 50 lb of a flour-sugar mixture that contains ten times as much flour as sugar. What are the possible weights of flour the baker can use? between about 36.4 lb and 45.5 lb flour

27. Between 15,000 yd^3 and 16,000 yd^3 of earth must be trucked away from a construction site. The trucks can remove 1000 yd^3 per day, and 10,500 yd^3 has already been removed. How many days are needed? between $4\frac{1}{2}$ and $5\frac{1}{2}$ days

28. By how much should a machinist decrease the length of a rod that is 4.78 cm long if the length must be within 0.02 cm of 4.5 cm? between 0.26 cm and 0.30 cm

B Apply Your Skills

Solve each inequality. Graph the solution. See margin for graphs.

29. $2 - 3z \geq 7(8 - 2z) + 12$ $z \geq 6$

30. $17 - 2y \leq 5(7 - 3y) - 15$ $y \leq \frac{3}{13}$

31. $\frac{2}{3}(x - 12) \leq x + 8$ $x \geq -48$

32. $\frac{3}{5}(x - 12) > x - 24$ $x < 42$

33. $3[4x - (2x - 7)] < 2(3x - 5)$ no solutions

34. $6[5y - (3y - 1)] \geq 4(3y - 7)$ All real numbers are solutions.

35. Writing Write a problem that can be solved using the inequality $x + 0.5x \leq 60$. See back of book.

36. Geometry The sum of the lengths of any two sides of a triangle is greater than the length of the third side. In $\triangle ABC$, $BC = 4$ and $AC = 8 - AB$. What can you conclude about AB? $2 < AB < 6$

37. a. Error Analysis Suppose a classmate writes $y \leq 20$ as the solution of $\frac{1}{2}(y - 16) \geq y + 2$. Prove that your classmate's answer is wrong by checking a number that is less than 20. Choose a number that makes the computation easy.

b. Solve $\frac{1}{2}(y - 16) \geq y + 2$. a. See margin p. 31. b. $y \leq -20$

38. Construction A contractor estimated that her expenses for a construction project would be between $700,000 and $750,000. She has already spent $496,000. How much more can she spend and remain within her estimate? between $204,000 and $254,000

Justifying Steps **Justify each step by identifying the property used.**

39.
$3x \leq 4(x - 1) - 8$
$3x \leq 4x - 4 - 8$ Dist. Prop.
$3x \leq 4x - 12$ arithmetic
$-x \leq -12$ Subt. Prop. of Ineq.
$x \geq 12$ Mult. Prop. of Ineq.

40.
$\frac{1}{2}(y + 3) > \frac{1}{3}(4 - y)$ Mult. Prop.
$3(y + 3) > 2(4 - y)$ of Ineq.
$3y + 9 > 8 - 2y$ Dist.Prop.
$5y + 9 > 8$ Add. Prop. of Ineq.
$5y > -1$ Subt. Prop. of Ineq.
$y > -0.2$ Div. Prop. of Ineq.

Real-World Connection

Careers To bid on a job, a construction contractor must consider all the costs of running a business as well as the costs of materials and labor.

Solve each compound inequality. Graph the solutions.

41–49. See margin p. 31 for graphs.

41. $-6 < 2x - 4 < 12$ $-1 < x < 8$

42. $11 < 3y + 2 < 20$ $3 < y < 6$

43. $-18 > 4x - 3 > -15$ no solutions

44. $36 \geq 1 - 5z > -21$ $-7 \leq z < 4\frac{2}{5}$

45. $5a - 4 > 16$ or $3a + 2 < 17$ All real numbers are solutions.

46. $6b + 3 < 15$ or $4b - 2 > 18$ $b < 2$ or $b > 5$

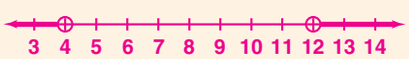
47. $6c \leq 18$ or $-5c \leq 15$ All real numbers are solutions.

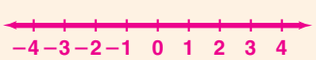
48. $8d < -64$ and $5d > 25$ no solutions

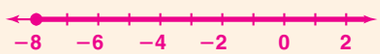
49. $4x \leq 12$ or $-7x \leq 21$ All real numbers are solutions.

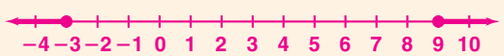
50. $15x > 30$ and $18x < -36$ no solutions

pages 29–31 Exercises

22. 

23. 

24. 

25. 

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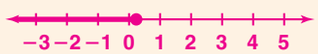
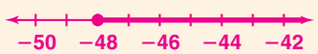
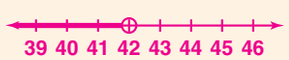
30. 

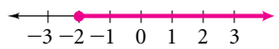
31. 

32. 

Challenge

Open-Ended **Write an inequality with a solution that matches the graph. At least two steps should be needed to solve your inequality.** Answers may vary. Sample: $-3x + 1 > 4$

51.



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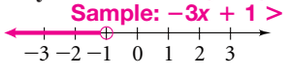

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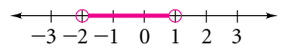

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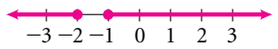


55. Critical Thinking Consider the compound inequality $x < 8$ and $x > a$.

a. Are there any values of a such that all real numbers are solutions of the compound inequality? If so, what are they? **no**

b. Are there any values of a such that no real numbers are solutions of the compound inequality? If so, what are they? **yes; values of *a* that are 8 or greater**

c. Repeat parts (a) and (b) for the compound inequality $x < 8$ or $x > a$. **a. yes; values of *a* that are less than 8**
b. no

Online Homework Help
Visit: PHSchool.com
Web Code: age-0104

51. Answers may vary. Sample: $2x - 7 \geq -11$

53. Answers may vary. Sample: $-9 < 5x + 1 < 6$

54. Answers may vary. Sample: $2x + 4 \leq 0$ or $-3x - 3 \leq 0$

Test Prep

Multiple Choice

56. Which of the following statements are true? **C**

I. $-(-6) = 6$ and $-(-4) > -4$
II. $-(-4) < 4$ or $-10 > 10 - 10$
III. $5 + 6 = 11$ or $9 - 2 = 11$
IV. $17 > 2$ or $6 < 9$

A. I and II only
B. I, II, and III only
C. I, III, and IV only
D. III and IV only

57. What is the solution of the inequality $8 - 3x < -3(1 + x) + 1$? **G**

F. all real numbers **G.** no real numbers **H.** $x > \frac{2}{3}$ **J.** $x < -\frac{11}{6}$

58. What is the solution of the compound inequality $2 < 2(x + 4) < 18$? **C**

A. all real numbers
B. no real numbers
C. $-3 < x < 5$
D. $-4 < x < 5$

59. What is the solution of the compound inequality $\frac{x}{2} - 4 > 0$ or $\frac{x}{2} + 1 < 0$? **J**

F. all real numbers
G. no real numbers
H. $x > 6$ or $x < 0$
J. $x > 8$ or $x < -2$

Short Response

60. What is the maximum number of 3- to 5-min songs that fill a 90-min CD? What is the minimum number? Explain your reasoning. **See margin.**

Extended Response

61. Fill each box with the word *and* or *or*, so that the solution of one compound inequality is *all real numbers* and the solution of the other is *no real numbers*. Justify each step of your solution. **See back of book.**

$x + 5 > 0$ ☐ $x - 3 < 0$ $\quad$ $x + 5 < 0$ ☐ $x + 5 > 0$

Mixed Review

Lesson 1-3 **Solve each equation. Check your answers.**

Go for Help

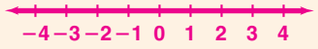
62. $7x - 6(11 - 2x) = 10$ **4**
63. $10x - 7 = 2(13 + 5x)$ **no solution**
64. $4y - \frac{1}{10} = 3y + \frac{4}{5}$ $\frac{9}{10}$
65. $0.4x + 1.18 = -3.1(2 - 0.01x)$ **−20**

Lesson 1-2 **Simplify each expression.**

66. $(2a - 4) + (5a + 9)$ **$7a + 5$**
67. $3(x + 3y) - 5(x - y)$ **$-2x + 14y$**
68. $\frac{1}{3}(b + 12) - \frac{1}{4}(b + 12)$ $\frac{1}{12}b + 1$
69. $0.4(k - 0.1) + 0.5(3.3 - k)$ **$1.61 - 0.1k$**

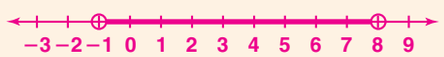
Online lesson quiz, PHSchool.com, Web Code: aga-0104

34.



37a. 0 makes $y \leq 20$ true, but it does not make $\frac{1}{2}(y - 16) \geq y + 2$ true.

41.



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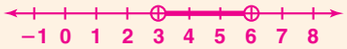

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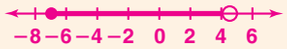

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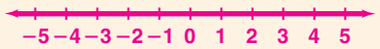
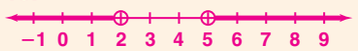


Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 51
- Test-Taking Strategies, p. 46
- Test-Taking Strategies with Transparencies

46.



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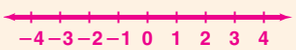

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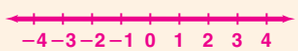


60. [2] The maximum number of songs can be recorded when the songs are short, so the maximum number of songs is $\frac{90 \text{ min}}{3 \text{ min per song}} = 30$ songs; the minimum number of songs can be recorded when the songs are long, so the minimum number of songs is $\frac{90 \text{ min}}{5 \text{ min per song}} = 18$ songs.

[1] provides 30 and 18 but not the explanation

1-5 Absolute Value Equations and Inequalities

What You'll Learn
- To solve absolute value equations
- To solve absolute value inequalities

. . . And Why
To write specifications for a basketball, as in Example 6

Check Skills You'll Need
GO for Help Lessons 1-3 and 1-4

Solve each equation.

1. $5(x - 6) = 40$ 14 **2.** $5b = 2(3b - 8)$ 16 **3.** $2y + 6y = 15 - 2y + 8$ 2.3

Solve each inequality.

4. $4x + 8 > 20$ $x > 3$ **5.** $3a - 2 \geq a + 6$ $a \geq 4$ **6.** $4(t - 1) < 3t + 5$ $t < 9$

New Vocabulary • absolute value • extraneous solution • tolerance

1 Absolute Value Equations

The **absolute value** of a number is its distance from zero on the number line and distance is nonnegative. So the absolute value of a negative number such as -5 is its opposite, $-(-5)$. For $x < 0$, $|x| = -x$.

Key Concepts

Definition Algebraic Definition of Absolute Value

- If $x \geq 0$, then $|x| = x$.
- If $x < 0$, then $|x| = -x$.

An absolute value equation such as $|2y - 4| = 12$ has two solutions, since the expression $2y - 4$ can equal 12 or -12.

1 EXAMPLE Solving Absolute Value Equations

Solve $|2y - 4| = 12$.

$|2y - 4| = 12$

$2y - 4 = 12$	or	$2y - 4 = -12$	The value of $2y - 4$ can be 12 or -12 since $\|12\|$ and $\|-12\|$ both equal 12.
$2y = 16$	|	$2y = -8$	Add 4 to each side of both equations.
$y = 8$	or	$y = -4$	Divide each side of both equations by 2.

Check $|2y - 4| = 12$

$|2(8) - 4| \stackrel{?}{=} 12$ $\quad$ $|2(-4) - 4| \stackrel{?}{=} 12$

$|12| = 12$ ✓ $\quad$ $|-12| = 12$ ✓

Quick Check
1 Solve $|3x + 2| = 7$. Check your answer. $\frac{5}{3}$, -3

You will find it easier to solve a multi-step absolute value equation if you first isolate the absolute value expression on one side of the equation.

Differentiated Instruction Solutions for All Learners

Special Needs L1
Apply the idea from the Error Prevention in Example 4 to $|2x + 6| < 8$ in Example 5 to help students understand the compound inequality.

learning style: verbal

Below Level L2
Draw a number line on the board, using inches or centimeters to mark the numbers. Measure the distance of positive and negative numbers from zero.

learning style: visual

1-5

1. Plan

Objectives
1 To solve absolute value equations
2 To solve absolute value inequalities

Examples
1 Solving Absolute Value Equations
2 Solving Multi-Step Absolute Value Equations
3 Checking for Extraneous Solutions
4 Solving Absolute Value Inequalities, $|A| \geq b$
5 Solving Absolute Value Inequalities, $|A| < b$
6 Real-World Connection

Professional Development

Math Background

To solve an equation that involves absolute value, it is possible to rewrite the given equation as two equations by first applying the algebraic definition of absolute value. However, first isolating the absolute value expression on one side of the equal sign and understanding that there may be two solutions leads to an approach that involves less algebraic manipulation.

More Math Background: p. 2D

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Solving Equations
Lesson 1-3: Examples 1, 2
Extra Skills and Word Problems
Practice, Ch. 1

Solving Inequalities
Lesson 1-4: Examples 1, 2
Extra Skills and Word Problems
Practice, Ch. 1

2. Teach

Guided Instruction

1 EXAMPLE Auditory Learners

Help students understand that $|a - b|$ represents the distance between a and b, where a and b are real numbers. Then, tell students that $|2y - 4| = 12$ can be read as "the distance between twice a number and 4 is 12." Help students use this statement to draw a number line and determine the solutions of the equation.

2 EXAMPLE Math Tip

It will be helpful to students to stress that $4w - 1$ can be 5 or -5.

3 EXAMPLE Teaching Tip

Point out that this example makes clear why it is always a good idea to check possible solutions to see whether they really *are* solutions.

PowerPoint

Additional Examples

1. Solve $|15 - 3x| = 6$. **3, 7**
2. Solve $4 - 2|x + 9| = -5$. **−13.5, −4.5**
3. Solve $|3x - 4| = -4x - 1$. **−5**

2 EXAMPLE Solving Multi-Step Absolute Value Equations

Solve $3|4w - 1| - 5 = 10$.

$3|4w - 1| - 5 = 10$

$3|4w - 1| = 15$ Add 5 to each side.

$|4w - 1| = 5$ Divide each side by 3.

$4w - 1 = 5$ or $4w - 1 = -5$ Rewrite as two equations.

$4w = 6$ | $4w = -4$ Add 1 to each side of both equations.

$w = \frac{3}{2}$ or $w = -1$ Divide each side of both equations by 4.

Check $3|4w - 1| - 5 = 10$

$3|4(\frac{3}{2}) - 1| - 5 \stackrel{?}{=} 10$

$3|5| - 5 \stackrel{?}{=} 10$

$10 = 10$ ✓

$3|4w - 1| - 5 = 10$

$3|4(-1) - 1| - 5 \stackrel{?}{=} 10$

$3|-5| - 5 \stackrel{?}{=} 10$

$10 = 10$ ✓

Quick Check 2 Solve $2|3x - 1| + 5 = 33$. Check your answer. $-\frac{13}{3}, 5$

The equation $|2x + 7| = -2$ has no solution because $|2x + 7|$ cannot be negative. It is important to check possible solutions in the original equation. One or more may be extraneous solutions.

Key Concepts

Definition **Extraneous Solution**

An **extraneous solution** is a solution of an equation derived from an original equation that is not a solution of the original equation.

3 EXAMPLE Checking for Extraneous Solutions

Solve $|2x + 5| = 3x + 4$.

$|2x + 5| = 3x + 4$

$2x + 5 = 3x + 4$ or $2x + 5 = -(3x + 4)$ Rewrite as two equations.

$-x = -1$ | $2x + 5 = -3x - 4$ Solve each equation.

$x = 1$ | $5x = -9$

$x = 1$ or $x = -\frac{9}{5}$

Check $|2x + 5| = 3x + 4$

$|2(1) + 5| \stackrel{?}{=} 3(1) + 4$

$|7| \stackrel{?}{=} 7$

$7 = 7$ ✓

$|2x + 5| = 3x + 4$

$|2(-\frac{9}{5}) + 5| \stackrel{?}{=} 3(-\frac{9}{5}) + 4$

$|\frac{7}{5}| \stackrel{?}{=} -\frac{7}{5}$

$\frac{7}{5} \neq -\frac{7}{5}$

The only solution is 1. $-\frac{9}{5}$ is an extraneous solution.

Vocabulary Tip

Extraneous is pronounced ek-STRAY-nee-us.

Quick Check 3 **a.** Solve $|2x + 3| = 3x + 2$. Check for extraneous solutions. **1**

b. Solve $|x| = x - 1$. Check for extraneous solutions. **no solutions**

c. Critical Thinking Find a value for a such that $|x| = x + a$ has exactly one solution. **Answers may vary: any positive real number**

Differentiated Instruction Solutions for All Learners

Advanced Learners L4

All sports use specifications such as diameter of carburetor restrictor plates or width of playing field. Find and write such data as an absolute value inequality.

learning style: verbal

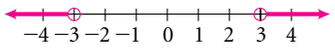
English Language Learners ELL

Help students with the pronunciation and the meaning of *extraneous*. As an example of its common meaning, discuss scenes that are extraneous to the main action of a play or a television show.

learning style: verbal

2 Absolute Value Inequalites

If $|x| > 3$, then x is more than 3 units from 0 on the number line.



This is also the graph of $x < -3$ or $x > 3$. So the absolute value inequality $|x| > 3$ can be rewritten as the compound inequality $x < -3$ or $x > 3$.

4 EXAMPLE Solving Absolute Value Inequalities, $|A| \geq b$

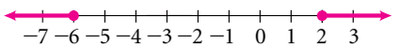
Solve $|3x + 6| \geq 12$. Graph the solution.

$|3x + 6| \geq 12$

$3x + 6 \leq -12$ or $3x + 6 \geq 12$ **Rewrite as a compound inequality.**

$3x \leq -18$ | $3x \geq 6$

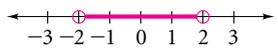
$x \leq -6$ or $x \geq 2$



Graph the solution.

Quick Check 4 Solve $|2x - 3| > 7$. Graph the solution. $x < -2$ or $x > 5$

If $|x| < 2$, then x is less than 2 units from 0 on the number line.



This is also the graph of $-2 < x < 2$. So the absolute value inequality $|x| < 2$ can be written as the compound inequality $-2 < x < 2$.

Key Concepts

Properties — Absolute Value Inequalities

Let k represent a positive real number.

$\|x\| \geq k$	is equivalent to	$x \leq -k$ or $x \geq k$.
$\|x\| \leq k$	is equivalent to	$-k \leq x \leq k$.

When an absolute value is combined with other operations, first isolate the absolute value expression on one side of the inequality.

5 EXAMPLE Solving Absolute Value Inequalities, $|A| < b$

Solve $3|2x + 6| - 9 < 15$. Graph the solution.

GO for Help

To review the properties of inequalities, go to p. 26.

$3|2x + 6| - 9 < 15$

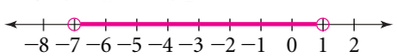
$3|2x + 6| < 24$ **Isolate the absolute value expression. Add 9 to each side.**

$|2x + 6| < 8$ **Divide each side by 3.**

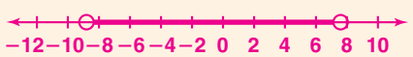
$-8 < 2x + 6 < 8$ **Rewrite as a compound inequality.**

$-14 < 2x < 2$ **Solve for x.**

$-7 < x < 1$



Quick Check 5 Solve $|5z + 3| - 7 < 34$. Graph the solution. $-8\frac{4}{5} < z < 7\frac{3}{5}$ See margin for graph.

5. 

Guided Instruction

4 EXAMPLE Error Prevention

Encourage students to check that the solutions indicated by their graphs are reasonable. Have students pick a few points in the solution sets and substitute them into the original equation to make certain that the resulting statements are true.

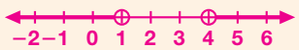
6 EXAMPLE Teaching Tip

You may find it helpful to review Example 6 from Lesson 1-4.

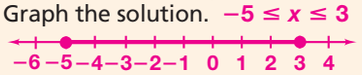
PowerPoint

Additional Examples

4 Solve $|2x - 5| > 3$. Graph the solution. $x < 1$ or $x > 4$



5 Solve $-2|x + 1| + 5 \geq -3$. Graph the solution. $-5 \leq x \leq 3$



6 The area A in square inches of a square photo is required to satisfy $8.5 \leq A \leq 8.9$. Write this requirement as an absolute value inequality. $|A - 8.7| \leq 0.2$

Resources

- Daily Notetaking Guide 1-5 L3
- Daily Notetaking Guide 1-5—Adapted Instruction L1

Closure

Have students describe the procedure used to solve an absolute value equation or inequality. **Isolate the absolute value expression on one side of the equation or inequality. Next, rewrite the result as two equations or as a compound inequality. Then, solve.**

3. Practice

Assignment Guide

1 A B	1-15, 34-43
2 A B	16-33, 44-60
C Challenge	61-66
Test Prep	67-72
Mixed Review	73-83

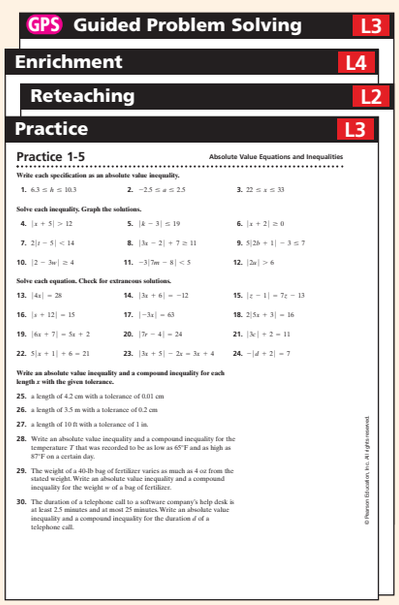
Connection to Meteorology

Exercise 54 Meteorologists use experimental probability based on previous weather patterns to make forecasts. Students will learn about experimental probability in Lesson 1-6.

Diversity

Exercise 55 Students who are not familiar with sports, may not realize that there are regulations that govern the size of basketballs for men and women. Discuss the regulations and the reasons behind them.

Differentiated Instruction Resources


GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Practice L3

Practice 1-5 Absolute Value Equations and Inequalities

Write each specification as an absolute value inequality.

1. $6.3 \le h \le 10.3$
2. $-2.5 \le a \le 2.5$
3. $22 \le x \le 33$

Solve each inequality. Graph the solutions.

4. $|x + 5| > 12$
5. $|k - 3| \le 19$
6. $|x + 2| \ge 0$
7. $2|t - 5| < 14$
8. $|3x - 2| + 7 \ge 11$
9. $5|2b + 1| - 3 \le 7$
10. $|2 - 3w| \ge 4$
11. $-3|7m - 8| < 5$
12. $|2a| > 6$

Solve each equation. Check for extraneous solutions.

13. $|4x| = 28$
14. $|3z + 6| = -12$
15. $|z - 1| = 7z - 13$
16. $|x + 12| = 15$
17. $|-3x| = 63$
18. $2|5x + 3| = 16$
19. $|6s + 7| = 5s + 2$
20. $|7r - 4| = 24$
21. $|3c| + 2 = 11$
22. $5|s + 1| + 6 = 21$
23. $|3s + 5| - 2s = 3s + 4$
24. $-|d + 2| = 7$

Write an absolute value inequality and a compound inequality for each length x with the given tolerance.

25. a length of 4.2 cm with a tolerance of 0.01 cm
26. a length of 3.5 m with a tolerance of 0.2 cm
27. a length of 10 ft with a tolerance of 1 in.
28. Write an absolute value inequality and a compound inequality for the temperature T that was recorded to be as low as 65°F and as high as 87°F on a certain day.
29. The weight of a 40-lb bag of fertilizer varies as much as 4 oz from the stated weight. Write an absolute value inequality and a compound inequality for the weight w of a bag of fertilizer.
30. The duration of a telephone call to a software company's help desk is at least 2.5 minutes and at most 25 minutes. Write an absolute value inequality and a compound inequality for the duration d of a telephone call.

You can use absolute value inequalities and compound inequalities to specify allowable ranges in measurements. The difference between a desired measurement and its maximum and minimum allowable values is the tolerance. The **tolerance** equals one half of the difference between the maximum and the minimum values.

For example, if a manufacturing specification calls for a dimension d of 10 cm with a tolerance of 0.1 cm, then the allowable difference between d and 10 is less than or equal to 0.1. This specification can be expressed in the following ways.

$\lvert d - 10 \rvert \le 0.1$	absolute value inequality
$d - 10 \le 0.1$ and $d - 10 \ge -0.1$	equivalent compound inequality
$-0.1 \le d - 10 \le 0.1$	equivalent compound inequality
$9.9 \le d \le 10.1$	simplified compound inequality

6 EXAMPLE **Real-World Connection**

Basketball The specification for the circumference C in inches of a basketball for men is $29.5 \le C \le 30$. Write the specification as an absolute value inequality.

$\frac{30 - 29.5}{2} = \frac{0.5}{2} = 0.25$	Find the tolerance.
$\frac{29.5 + 30}{2} = 29.75$	Find the average of the maximum and minimum values.
$-0.25 \le C - 29.75 \le 0.25$	Write an inequality.
$\lvert C - 29.75 \rvert \le 0.25$	Rewrite as an absolute value inequality.

Quick Check

6 The specification for the circumference C in inches of a basketball for junior high school is $27.75 \le C \le 28.5$. Write the specification as an absolute value inequality.
$|C - 28.125| \le 0.375$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (pages 33, 34)

GO for Help

Solve each equation. Check your answers.

1. $|3x| = 18$ **−6, 6**
2. $|-4x| = 32$ **−8, 8**
3. $|x - 3| = 9$ **−6, 12**
4. $2|3x - 2| = 14$ **3, $-\frac{5}{3}$**
5. $|3x + 4| = -3$ **no solution**
6. $|2x - 3| = -1$ **no solution**
7. $|x + 4| + 3 = 17$ **−18, 10**
8. $|y - 5| - 2 = 10$ **−7, 17**
9. $|4 - z| - 10 = 1$ **−7, 15**

Example 3 (page 34)

Solve each equation. Check for extraneous solutions.

10. $|x - 1| = 5x + 10$ **$-\frac{3}{2}$**
11. $|2z - 3| = 4z - 1$ **$\frac{2}{3}$**
12. $|3x + 5| = 5x + 2$ **$\frac{3}{2}$**
13. $|2y - 4| = 12$ **−4, 8**
14. $3|4w - 1| - 5 = 10$ **$-1, \frac{3}{2}$**
15. $|2x + 5| = 3x + 4$ **1**

Example 4 (page 35)

Solve each inequality. Graph the solution. **16–23. See margin pp. 36–37.**

16. $|x + 3| > 9$
17. $|x - 5| \ge 8$
18. $|y - 3| \ge 12$
19. $|2x + 1| \ge -9$
20. $3|2x - 1| \ge 21$
21. $|3z| - 4 > 8$

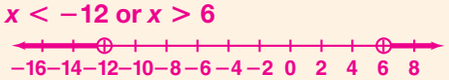
Example 5 (page 35)

24–27. See back of book.

22. $3|y - 9| < 27$
23. $|6y - 2| + 4 < 22$
24. $|3x - 6| + 3 < 15$
25. $\frac{1}{4}|x - 3| + 2 < 1$
26. $4|2w + 3| - 7 \le 9$
27. $3|5t - 1| + 9 \le 23$

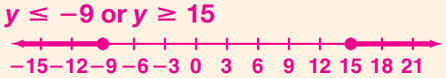
pages 36–38 Exercises

16. $x < -12$ or $x > 6$



17. $x \le -3$ or $x \ge 13$

18. $y \le -9$ or $y \ge 15$



19. all real numbers are solutions.

Example 6 (page 36)

Write each specification as an absolute value inequality.

28. $1.3 \le h \le 1.5$ $|h - 1.4| \le 0.1$

29. $50 \le k \le 51$ $|k - 50.5| \le 0.5$

30. $27.25 \le C \le 27.75$ $|C - 27.5| \le 0.25$

31. $50 \le b \le 55$ $|b - 52.5| \le 2.5$

32. $1200 \le m \le 1300$ $|m - 1250| \le 50$

33. $0.1187 \le d \le 0.1190$ $|d - 0.11885| \le 0.00015$

B Apply Your Skills

Solve each equation.

GPS **34.** $-|4 - 8b| = 12$ no solutions

35. $4|3x + 4| = 4x + 8$ $-\frac{3}{2}, -1$

36. $|3x - 1| + 10 = 25$ $-\frac{14}{3}, \frac{16}{3}$

37. $\frac{1}{2}|3c + 5| = 6c + 4$ $-\frac{1}{3}$

38. $5|6 - 5x| = 15x - 35$ no solutions

39. $7|8 - 3h| = 21h - 49$ $\frac{5}{2}$

40. $2|3x - 7| = 10x - 8$ $\frac{11}{8}$

41. $6|2x + 5| = 6x + 24$ $-1, -3$

42. $\frac{1}{4}|4x + 7| = 8x + 16$ $-\frac{71}{36}$

43. $\frac{2}{3}|3x - 6| = 4(x - 2)$ 2

GO Online Homework Help

Visit: PHSchool.com
Web Code: age-0105

Solve each inequality. Graph the solutions. 44–50. See back of book.

44. $|3x - 4| + 5 \le 27$

45. $|2x + 3| - 6 \ge 7$

46. $-2|x + 4| < 22$

47. $2|4t - 1| + 6 > 20$

48. $|3z + 15| \ge 0$

49. $|-2x + 1| > 2$

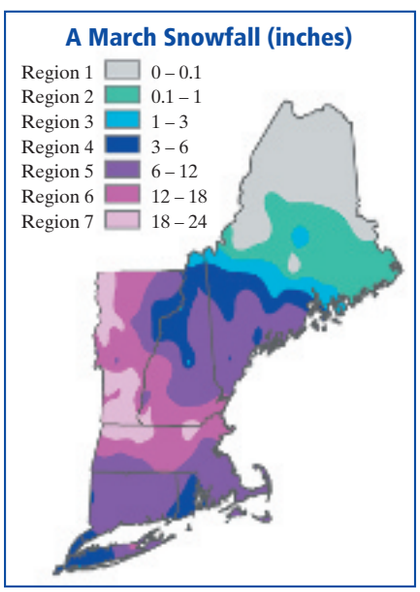
50. $\frac{1}{9}|5x - 3| - 3 \ge 2$

51. $\frac{1}{11}|2x - 4| + 10 \le 11$

52. $|\frac{x - 3}{2}| + 2 < 6$

53. $|\frac{x + 5}{3}| - 3 > 6$

51–53. See margin p. 38.



Source: Northeast River Forecast Center/NOAA

54. Meteorology Write a compound inequality and an absolute value inequality for the snowfall in regions 1, 2, 3, and 4 in the figure at the left. See back of book.

55. Multiple Choice The circumference of a basketball for college women must be from 28.5 in. to 29.0 in. Which absolute value inequality best represents the circumference of the ball? C

(A) $|C - 0.25| \ge 28.5$ (B) $|C - 0.25| \le 29.0$

(C) $|C - 28.75| \le 0.25$ (D) $|C - 28.75| \ge 0.25$

56. Writing Describe the differences in the graphs of $|x| < a$ and $|x| > a$, where a is a positive real number. See back of book.

57. Open-Ended Write an absolute value inequality for which every real number is a solution. Write an absolute value inequality that has no solution.
Answers may vary. Sample: $|x - 1| \ge 0$; $|x| < -5$

Write an absolute value inequality and a compound inequality for each length x with the given tolerance.

58. a length of 36.80 mm with a tolerance of 0.05 mm $|x - 36.8| \le 0.05$; $36.75 \le x \le 36.85$

59. a length of 9.55 mm with a tolerance of 0.02 mm $|x - 9.55| \le 0.02$; $9.53 \le x \le 9.57$

60. a length of 100 yd with a tolerance of 4 in. x is in inches: $|x - 3600| \le 4$; $3596 \le x \le 3604$.

C Challenge

Solve each equation for x.

61. $|ax| - b = c$ $\frac{b + c}{a}, \frac{b + c}{a}$

62. $|cx - d| = ab$ $\frac{ab + d}{c}, \frac{-ab + d}{c}$

63. $a|bx - c| = d$ $\frac{ac + d}{ab}, \frac{ac - d}{ab}$

Graph each solution. 64–66. See back of book.

64. $|x| \ge 5$ and $|x| \le 6$

65. $|x| \ge 6$ or $|x| < 5$

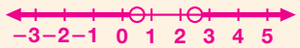
66. $|x - 5| \le x$

4. Assess & Reteach

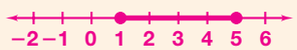
PowerPoint

Lesson Quiz

1. Solve $|3x + 1| = 4$. $-\frac{5}{3}, 1$
2. Solve $|-2x + 3| + 7 > 9$. Graph the solution. $x < \frac{1}{2}$ or $x > \frac{5}{2}$



3. Solve $|4x - 12| \le 8$. $1 \le x \le 5$



4. A machinist is to drill a hole with diameter D inches, that satisfies $0.16 \le D \le 0.19$. Express the specification using an absolute value inequality. $|D - 0.175| \le 0.015$

Alternative Assessment

Have students work in pairs. Each student writes two absolute value equations and two absolute value inequalities. Each student then solves his or her partner's equations and inequalities. Then, they check one another's work.

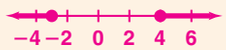
Test Prep

Resources

For additional practice with a variety of test item formats:

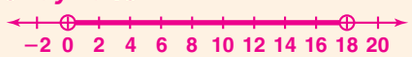
- Standardized Test Prep, p. 51
- Test-Taking Strategies, p. 46
- Test-Taking Strategies with Transparencies

20. $x \le -3$ or $x \ge 4$

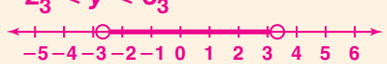


21. $z < -4$ or $z > 4$

22. $0 < y < 18$



23. $-2\frac{2}{3} < y < 3\frac{1}{3}$



Test Prep

Multiple Choice

67. Which number is a solution of $|x - 3| = x - 3$? **D**

A. -3 **B.** 0 **C.** 1 **D.** 3

68. What is the solution of the inequality $\left|\frac{3 - x}{2}\right| < 4$? **F**

F. $-5 < x < 11$ **G.** $-11 > x > -5$ **H.** $5 < x < 11$ **J.** $11 > x > -1$

69. Which of the following inequalities have the same solutions? **C**

I. $|5x - 7| \leq 8$ **II.** $-8 \leq 5x - 7$ or $5x - 7 \leq 8$

III. $8 \leq 5x - 7$ and $5x - 7 \geq -8$ **IV.** $-8 \leq 5x - 7$ and $5x - 7 \leq 8$

A. I and II **B.** I and III **C.** I and IV **D.** I, III, and IV

70. Which number is a solution of $|9 - x| = 9 + x$? **G**

F. -3 **G.** 0 **H.** 3 **J.** 6

Short Response

71. Find all the integers that are solutions of $|x - 3| \leq 5$. Show your work. **See margin.**

Extended Response

72. Solve $3|2x - 4| + 5 < 41$. Justify each step of your solution. **See margin.**

Mixed Review

Lesson 1-4

Solve each inequality. Graph the solution. **See margin for graphs.**

73. $5y - 10 < 20$ **$y < 6$**

74. $-5(4s + 1) < 23$ **$s > -\frac{7}{5}$**

75. $4a + 6 \geq 2a + 14$ **$a \geq 4$**

76. $0.5x + 5 \geq x - 1$ **$x \leq 12$**

77. $3(4x - 1) \geq 2(4 - x)$ **$x \geq \frac{11}{14}$**

78. $4(3t + 2) \leq 43 + 7t$ **$t \leq 7$**

Lesson 1-2

Evaluate each expression for the given value.

79. $3|4x - 6| - 2x^2$, for $x = -3$ **36**

80. $\frac{5r - r^2}{1 - 4r}$, for $r = 4$ **$-\frac{4}{15}$**

Lesson 1-1

Name the property of real numbers illustrated by each of the following.

81. $16x + (-16x) = 0$ **Inverse Prop. of Add.**

82. 5π is a real number. **Closure Prop. of Mult.**

83. $4(x - 9) = (x - 9)4$ **Comm. Prop. of Mult.**

Checkpoint Quiz 2 — Lessons 1-4 through 1-5

Solve each inequality. Graph the solution. **1–6. See back of book for graphs.**

1. $3x + 10 \leq 25$ **$x \leq 5$**

2. $8x + 15 > 15x - 24$ **$x < 5\frac{4}{7}$**

3. $5z > 2z - 18$ and $3 - 9z < 12$

4. $4w > 1 + 3w$ or $12w + 18 < 11w + 15$

5. $2|x + 4| \leq 22$ **$-15 \leq x \leq 7$**

6. $|2x| + 8 > 12$ **$x < -2$ or $x > 2$**

Solve each equation.

7. $7|3 - 2y| = 56$ **$-\frac{5}{2}, \frac{11}{2}$**

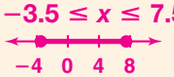
8. $\frac{1}{4}|4x + 2| = 1 - 2x$ **$\frac{1}{6}$**

9. Write and solve an inequality to find three consecutive whole numbers with a sum between 13 and 16. **$13 < 3n + 3 < 16$, $n = 4$**

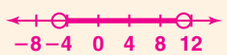
10. Geometry The length of any side of a triangle is less than the sum of the lengths of the other two sides. In $\triangle PQR$, $PR = RQ + 4$ and $RQ < 11$. Write and solve an inequality for PQ. **$PR - RQ < PQ < PR + RQ$; $4 < PQ < 26$**

pages 36–38 Exercises

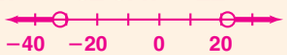
51. $-3.5 \leq x \leq 7.5$



52. $-5 < x < 11$



53. $x < -32$ or $x > 22$



71. [2] $|x - 3| \leq 5$
$-5 \leq x - 3 \leq 5$
$-2 \leq x \leq 8$

[1] only includes $-2 \leq x \leq 8$ and does not show work

72. [4] $3|2x - 4| + 5 < 41$
$3|2x - 4| < 36$
Subtr. Prop. of Ineq.
$|2x - 4| < 12$
Div. Prop. of Ineq.
$-12 < 2x - 4 < 12$
Def. of absolute value
$-8 < 2x < 16$
Add. Prop. of Ineq.
$-4 < x < 8$
Div. Prop. of Ineq.

[3] appropriate methods, but with one computational error

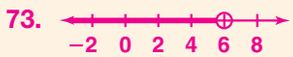
[2] does not include steps

[1] only includes final answer of $-4 < x < 8$, with no steps or justification of steps

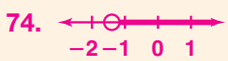
3. $z > -1$

4. $w > 1$ or $w < -3$

73.

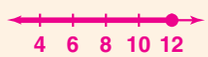


74.



75.

76.



77.

78.

1-6 Probability

What You'll Learn

- To find experimental probabilities
- To find theoretical probabilities

. . . And Why

To find the probabilities of inherited traits, as in Example 4

Check Skills You'll Need

GO for Help Skills Handbook page 870

Write each number as a percent.

1. $\frac{3}{8}$ 37.5%
2. $1\frac{5}{6}$ $183\frac{1}{3}$%
3. 0.0043 0.43%
4. $\frac{1}{400}$ 0.25%
5. 1.04 104%
6. 3 300%

New Vocabulary

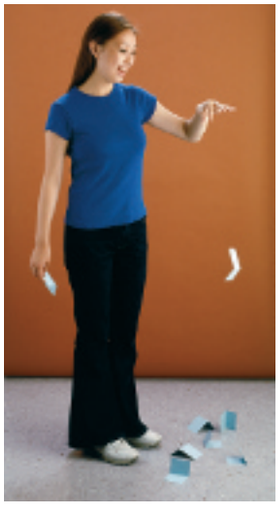
- experimental probability
- simulation
- sample space
- theoretical probability

1 Experimental Probability

Hands-On Activity: Experimental Probability

Fold an index card slightly off center, as shown at the left. When you drop the card from a height of several feet, how will it land?

1–5. Check students' work.



1. Drop the card 50 times. Record the number of times the card lands in each position.
2. What percent of the time does the card land on its short side? Find the percents for the other positions.
3. In what position is the card most likely to land? Least likely to land?
4. Suppose that you drop the card another 20 times. Predict how many times it will land in each position.
5. **a.** Drop the card another 20 times. Record your results.
   b. Compare the results with your prediction from Question 4. Are they close? How could you improve your prediction?

Differentiated Instruction Solutions for All Learners

Special Needs L1

Ask questions such as: *What is the probability you will guess the answer to a true-false question?* 50% *If there are four answers to a multiple-choice question, what is the probability that you can guess the correct answer?* 25%

learning style: verbal

Below Level L2

Discuss with students why a 50% chance of rain today and a 50% chance of rain tomorrow does not mean it is certain to rain.

learning style: verbal

1-6

1. Plan

Objectives

1 To find experimental probabilities
2 To find theoretical probabilities

Examples

1 Finding Experimental Probability
2 Using a Simulation
3 Finding Theoretical Probability
4 Real-World Connection
5 Finding Geometric Probability

Professional Development

Math Background

Experimental probabilities are calculated on the basis of data from experiments, actual or simulated. Given equally likely outcomes, the basis for calculating theoretical probability is being able to determine the number of ways that an event can occur within these outcomes. Comparisons of measures such as length and area are the basis of geometric probability.

More Math Background: p. 2D

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

Percent and Percent Applications
Skills Handbook: p. 870
Example 6

2. Teach

Guided Instruction

Hands-On Activity
You may want students to work on this Activity with a partner. The results that students obtain may be influenced by a variety of factors, such as how far from the end the card was folded.

1 EXAMPLE Diversity

Invite students who are more familiar with other sports, such as cricket, to suggest examples of experimental probabilities related to those sports.

2 EXAMPLE Alternative Method

Invite students to suggest other simulations such as spinning a two-section spinner.

Additional Examples

1 A player hit the bull's eye on a circular dartboard 8 times out of 50. Find the experimental probability that the player hits the bull's eye. **0.16, or 16%**

2 Describe a simulation you could use that involves flipping a coin to find the experimental probability of guessing exactly 2 answers out of 6 correctly on a true-false quiz. **Answers may vary. Sample: Let heads represent a correct answer. Flip the coin 6 times. Record the number of heads. Repeat 100 times. Divide the number of times you got 2 heads by 100.**

Probability measures how likely it is for an event to occur. You can express probabilities as percents (0% through 100%) or as real numbers (0 through 1).

The probability of an impossible event is 0 or 0%. The probability of a certain event, which must happen, is 1 or 100%.

When you gather data from observations, you can calculate an experimental probability. Each observation is called an experiment or a trial.

Key Concepts

Definition **Experimental Probability**

experimental probability of event $= P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$

1 EXAMPLE Finding Experimental Probability

Vocabulary Tip

Read $P(\text{hit})$ as "probability of a hit."

A baseball player got a hit 21 times in 60 at-bats. Find the experimental probability of his getting a hit.

$P(\text{hit}) = \frac{21}{60} = 0.35$, or 35%

Quick Check

1 A basketball player has made 32 free throws in 50 tries. What is the experimental probability of her making a free throw? $\frac{32}{50}$, **or 0.64, or 64%**

When actual trials are difficult to conduct, you can find experimental probabilities by using a **simulation,** which is a model of one or more events.

2 EXAMPLE Using a Simulation

Suppose you take a true-or-false quiz and guess four answers at random. What is an experimental probability that you will get at least three correct answers?

Step 1 Define how you will do the simulation.

- Generate random numbers on a calculator.
- Since you answer true or false at random, you have a 50% chance of guessing correctly on each question. So let half of the digits represent correct answers. For example, let even digits represent correct answers.
- Since there are four questions, group the random digits in groups of four. List 50 groups to represent taking the test 50 times.

rand
.8767044746
.7208728315
.6495877886
.3462435756
.8958229562
.4615208491

.1214849574
.5070685798
.5584414289
.9071019949
.1991062533
.4761581612
.4792397345

.6388889895
.5466561294
.2840659518
.2096111775
.7827769142
.7787959459
.6958584068

Step 2 Conduct the simulation. Underline groups with at least three even digits.

8767	0447	4672	0872	8315	6495	8778	8634	6243	5756
8958	2295	6246	1520	8491	1214	8495	7450	7068	5798
5584	4142	8990	7101	9949	1991	0625	3347	6158	1612
4792	3973	4563	8888	9895	5466	5612	9428	4065	9518
2096	1117	7578	2776	9142	7787	9594	5969	5858	4068

Step 3 Interpret the simulation. Since 15 of the 50 groups represent at least three correct answers, $P(\text{at least 3 correct}) = \frac{15}{50} = 0.3$.

This experimental probability of getting at least three correct answers is 30%.

Quick Check

2 What is the experimental probability of getting all four answers correct? $\frac{3}{50}$, **or 6%**

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
To estimate π, drop a toothpick onto parallel lines spaced a toothpick-length apart. Divide twice the number of drops by the number of lines hit. Discuss.

learning style: tactile

English Language Learners ELL
In Example 2, be sure that the students understand the meaning of *at random* as having no set pattern or design. If a student guessed "T, F, T, F, T, F, T, F" that would be a pattern. To guess at random could be, for example, "T, T, T, F, T, F, F, T".

learning style: verbal

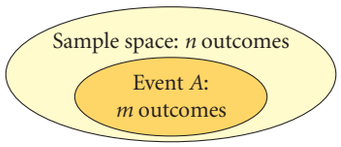
2 Theoretical Probability

When you roll a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6. The set of all possible outcomes is called the **sample space.** You can calculate theoretical probability as a ratio of outcomes.

Key Concepts

Definition **Theoretical Probability**

If a sample space has n equally likely outcomes and an event A occurs in m of these outcomes, then the **theoretical probablilty** of event A is $P(A) = \frac{m}{n}$.



3 EXAMPLE Finding Theoretical Probability

Find the theoretical probability of getting an even number when you roll a number cube.

The even outcomes are 2, 4, and 6.

3 outcomes result in an even number. → $\frac{3}{6}$ ← 6 equally likely outcomes are in the sample space.

$= \frac{1}{2}$

Quick Check 3 Find the theoretical probability of getting a prime number when you roll a number cube. $\frac{1}{2}$, or 50%

4 EXAMPLE Real-World Connection

Biology Fold your hands so your fingers interlace. Do you naturally place your left or right thumb on top? Placing your left thumb on top is a dominant genetic trait.

When a parent has both a dominant and a recessive gene, then the two genes are equally likely to be passed to a child. If you have one or two dominant genes, you normally place your left thumb on top.

Suppose a child has parents who both have just one dominant gene. What is the theoretical probability that the child will naturally place the left thumb on top?

Make a table. Let G represent the dominant gene (left thumb on top). Let g represent the recessive gene (right thumb on top).

		Gene from Mother	
		G	g
Gene from Father	G	GG	Gg
	g	Gg	gg

The sample space {GG, Gg, Gg, gg} contains four equally likely outcomes. Three outcomes have at least one G. So P(left thumb on top) $= \frac{3}{4}$.

The theoretical probability that the child will naturally place the left thumb on top is $\frac{3}{4}$, or 75%.

Quick Check 4 What is the theoretical probability that a child of the parents in Example 4 places the right thumb on top? $\frac{1}{4}$, or 25%

Guided Instruction

3 EXAMPLE Math Tip

Students need to understand that the theoretical probability of $\frac{1}{2}$ does not mean that, in a particular group of rolls, exactly half the rolls will result in an even number. Probability tells what is *likely* to happen, not what *must* happen.

4 EXAMPLE Career Note

Inherited traits are important in understanding many types of health problems. There are exciting career opportunities in health care and the pharmaceutical industry that are related to genetic research.

5 EXAMPLE Connection to Math

Students may be interested to know that simulations and ideas about geometric probability can be used to find approximate values of areas and important constants such as π.

PowerPoint

Additional Examples

3 Find the theoretical probability of rolling a multiple of 3 with a number cube. $\frac{1}{3}$

4 Brown is a dominant eye color for human beings. If a father and mother each carry a gene for brown eyes and a gene for blue eyes, what is the probability of their having a child with blue eyes? $\frac{1}{4}$

5 For the dartboard in Example 5, find the probability that a dart that lands at random on the dartboard hits the outer ring. $\frac{7}{16}$

Resources

- Daily Notetaking Guide 1-6 L3
- Daily Notetaking Guide 1-6—Adapted Instruction L1

Closure

Tell students to name two pieces of information that can be used to find the theoretical probability of an event. **number of possible outcomes, number of outcomes representing the event in**

3. Practice

Assignment Guide

1 A B 1-5, 21-23, 40

2 A B 6-20, 24-39, 41-42

C Challenge 43-45

Test Prep 46-51
Mixed Review 52-63

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 4, 14, 38, 40, 41.

Technology Tip

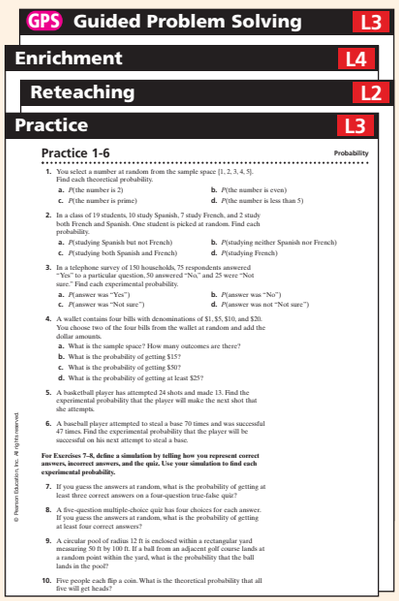
Exercises 3, 4 Students can enter the expression randInt(0,1,5) on the home screen of their graphing calculators using the catalog feature. Then they can press ENTER to generate a random sequence of five integers where each integer is either a 0 or 1. Repeatedly pressing ENTER will generate a new sequence each time. Students may wish to use this idea for their simulation.

Error Prevention!

Exercises 24–26 Students should not overlook the 31 students who take neither math nor science.

Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Practice L3



Practice 1-6 Probability

1. You select a number at random from the sample space {1, 2, 3, 4, 5}. Find each theoretical probability.
   a. P(the number is 2) b. P(the number is even)
   c. P(the number is prime) d. P(the number is less than 5)
2. In a class of 19 students, 10 study Spanish, 7 study French, and 2 study both French and Spanish. One student is picked at random. Find each probability.
   a. P(studying Spanish but not French) b. P(studying neither Spanish nor French)
   c. P(studying both Spanish and French) d. P(studying French)
3. In a telephone survey of 150 households, 75 respondents answered "Yes" to a particular question, 50 answered "No," and 25 were "Not sure." Find each experimental probability.
   a. P(answer was "Yes") b. P(answer was "No")
   c. P(answer was "Not sure") d. P(answer was not "Not sure")
4. A wallet contains four bills with denominations of $1, $5, $10, and $20. You choose two of the four bills from the wallet at random and add the dollar amounts.
   a. What is the sample space? How many outcomes are there?
   b. What is the probability of getting $15?
   c. What is the probability of getting $30?
   d. What is the probability of getting at least $25?
5. A basketball player has attempted 24 shots and made 13. Find the experimental probability that the player will make the next shot that she attempts.
6. A baseball player attempted to steal a base 70 times and was successful 47 times. Find the experimental probability that the player will be successful on his next attempt to steal a base.

For Exercises 7–8, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability.

7. If you guess the answers at random, what is the probability of getting at least three correct answers on a four-question true-false quiz?
8. A five-question multiple-choice quiz has five choices for each answer. If you guess the answers at random, what is the probability of getting at least four correct answers?
9. A circular pool of radius 12 ft is enclosed within a rectangular yard measuring 50 ft by 100 ft. If a ball from an adjacent golf course lands at a random point within the yard, what is the probability that the ball lands in the pool?
10. Five people each flip a coin. What is the theoretical probability that all five will get heads?

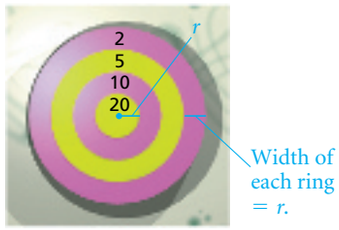
Sometimes you can use areas to find theoretical probability.

Online active math

For: Probability Activity
Use: Interactive Textbook, 1-6

5 EXAMPLE Finding Geometric Probability

Geometry Suppose that all the points on the circular dartboard shown at the right are equally likely to be hit by a dart you have thrown. Find the probability of scoring at least ten points.



$$P(\text{at least 10 points}) = \frac{\text{area of circle with radius } 2r}{\text{area of circle with radius } 4r}$$

$$= \frac{\pi(2r)^2}{\pi(4r)^2}$$

$$= \frac{4\pi r^2}{16\pi r^2} = \frac{1}{4}$$

The theoretical probability of scoring at least ten points is $\frac{1}{4}$, or 25%.

Quick Check 5 Use the dartboard from Example 5. Find each probability.

a. P(scoring 20 points) $\frac{1}{16}$, or 6.25%

b. P(scoring 5 points) $\frac{5}{16}$, or 31.25%

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 40)

GO for Help

1. A class tossed coins and recorded 161 heads and 179 tails. What is the experimental probability of heads? Of tails? $\frac{161}{340} \approx 47\%$; $\frac{179}{340} \approx 53\%$
2. Another class rolled number cubes. Their results are shown in the table. What is the experimental probability of rolling each number? See margin.

Number	1	2	3	4	5	6
Occurrences	42	44	45	44	47	46

Example 2 (page 40)

For Exercises 3–5, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability. 3–4. See margin pp. 42–43.

Graphing Calculator Hint

To generate random numbers, press MATH ◀ 1 ENTER.

3. If you guess the answers at random, what is the probability of getting at least two correct answers on a five-question true-or-false quiz?
4. If you guess the answers at random, what is the probability of getting at least three correct answers on a five-question true-or-false quiz ?
5. A five-question multiple-choice quiz has five choices for each answer. What is the probability of correctly guessing at random exactly one correct answer? Exactly two correct answers? Exactly three correct answers? (*Hint:* You could let any two digits represent correct answers, and the other digits represent wrong answers.) See back of book.

Example 3 (page 41)

A jar contains 30 red marbles, 50 blue marbles, and 20 white marbles. You pick one marble from the jar at random. Find each theoretical probability.

6. P(red) $\frac{3}{10}$, or 30%
7. P(blue) $\frac{1}{2}$, or 50%
8. P(not white) $\frac{4}{5}$, or 80%
9. P(red or blue) $\frac{4}{5}$, or 80%

pages 42–45 Exercises

2. the number 1: $\frac{21}{134}$, or about 15.7%; the number 2: $\frac{11}{67}$ or about 16.4%; the number 3: $\frac{45}{268}$ or about 16.8%; the number 4: $\frac{11}{67}$ or about 16.4%; the number 5: $\frac{47}{268}$ or about 17.5%; the number 6: $\frac{23}{134}$ or about 17.2%

3. Answers may vary. Sample: Generate random numbers between 0 and 1 using a graphing calculator. In each random number, examine the first five digits. Let even digits represent correct answers and odd digits incorrect answers. If there

10. $\frac{48}{125}$, or 38.4%

11. $\frac{19}{125}$, or 15.2%

12. $\frac{103}{125}$, or 82.4%

13. $\frac{14}{25}$, or 56%

A bag contains 36 red, 48 green, 22 yellow, and 19 purple blocks. You pick one block from the bag at random. Find each theoretical probability. 10–13. See left.

10. P(green) **11.** P(purple) **12.** P(not yellow)

13. P(green or yellow) **14.** P(yellow or not green) $\frac{77}{125}$, or 61.6%

Example 4 (page 41)

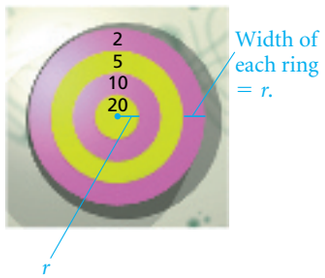
For each situation, find the sample space and the theoretical probability that a child will naturally place the left thumb on top. {*Gg*, *Gg*, *gg*, *gg*}; $\frac{1}{2}$, or 50%

15. The father has gene pair gg and the mother has Gg.

16. The father has gene pair gg and the mother has GG. {*Gg*, *Gg*, *Gg*, *Gg*}; 1, or 100%

Example 5 (page 42)

Geometry **Suppose that a dart lands at random on the dartboard shown at the right. Find each theoretical probability.** 17–20. See left.



17. $\frac{1}{16}$, or 6.25%

18. $\frac{3}{8}$, or 37.5%

19. $\frac{1}{4}$, or 25%

20. $\frac{3}{4}$, or 75%

17. The dart lands in the bull's-eye.

18. The dart lands in a green region.

19. The dart scores at least 10 points.

20. The dart scores less than 10 points.

B Apply Your Skills

21. The common interpretation of Murphy's Law is, If something can go wrong, it will. Assume that Murphy's Law applies to the following situations, and estimate each probability as either 0 or 1.
a. P(your dog chews up your homework after you've finished it) 1
b. P(your teacher accepts your excuse for not having your homework) 0

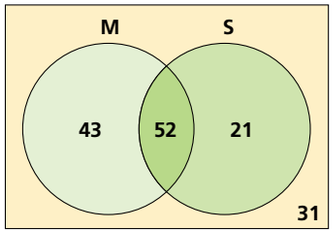
22. **Multiple Choice** A caterer knows that, on average, there will be one broken egg in every 3 cartons. A carton contains 12 eggs. The caterer plans to serve 1200 eggs at a breakfast. What is the best estimate for the number of cartons the caterer should buy? C

(A) 97 cartons (B) 100 cartons (C) 103 cartons (D) 112 cartons

23. Use the random number table at the left to simulate tossing a coin 50 times. Find the experimental probability that the outcome of a coin toss is heads. See margin.

Random Number Table		
31504	51648	40613
79321	80927	42404
15594	84675	68591
34178	00460	31754
49676	58733	00884
85400	72294	22551
22547	86066	93114
85211	07790	20890
21339	09414	51549
13843	18407	87043
34990	16214	46849
11390	01322	82656
45950	37521	77417

In a class of 147 students, 95 are taking math (M), 73 are taking science (S), and 52 are taking both math and science. One student is picked at random. Find each probability. 24–27. See left.



24. P(taking math or science or both)

25. P(not taking math)

26. P(taking math but not science)

27. P(taking neither math nor science)

Suppose you roll a number cube. Find each theoretical probability.

28. $P(5)$ $\frac{1}{6}$ **29.** P(an even number) $\frac{1}{2}$

30. P(a number less than 5) $\frac{2}{3}$ **31.** $P(8)$ 0

32. P(a number greater than 5) $\frac{1}{6}$ **33.** P(a number less than 8) 1

Exercise 36 If necessary, remind students that a prime number is a whole number greater than 1 whose only factors are 1 and itself.

Error Prevention!

Exercise 38 It is assumed that the number cubes are distinguishable (for example, of different colors). This means there are *two* ways, not just one, for a pair of numbers such as 2 and 5 to be rolled. It would be beneficial for students to list all the possibilities if they find it difficult to describe the sample space accurately in words.

English Language Learners ELL

Exercise 44 Be sure students understand that exchanges are distinct from area codes.

24. $\frac{116}{147}$, or 78.9%

25. $\frac{52}{147}$, or 35.4%

26. $\frac{43}{147}$, or 29.3%

27. $\frac{31}{147}$, or 21.1%

are two or more even digits, make a tally mark for that number. Do this 100 times. Find the total number of tally marks. This, as a percent, gives the experimental probability.

The simulated probability should be about 70%.

4. Answers may vary. Sample: Toss 5 coins. Keep a tally of the times 3 or more heads are tossed. (A head represents a correct answer.) Do this 100 times. The total number of tally marks, as a percent, gives the experimental probability. The simulated probability should be about 40%.

23. Answers may vary. Sample: Let odd digits represent heads and even digits represent tails. Use the first four rows of the table. The experimental probability of heads is $\frac{1}{2}$.

4. Assess & Reteach

PowerPoint

Lesson Quiz

1. A bowler rolled the ball 35 times and got 5 strikes. What is the experimental probability that the bowler gets a strike? $\frac{1}{7}$
2. Describe a simulation you could use that involves flipping a coin to find the experimental probability of guessing at least four correct answers on a five-question true or false quiz. **Answers may vary. Sample: Let heads represent a correct answer and tails represent an incorrect answer. Flip the coin five times. Record the number of heads. Repeat 100 times. Divide the number of times you got 4 or 5 heads by 100.**
3. What is the theoretical probability of rolling a sum less than 5 using two number cubes? $\frac{1}{6}$
4. Segments parallel to the sides are used to divide a square board 3 ft on each side into 9 equal-size smaller squares. If the board is in a level position and a grain of rice lands on the board at a random point, what is the probability that it lands on a corner section? $\frac{4}{9}$

Alternative Assessment

Have students work in groups of three. Suppose that each group is planning a vacation. From the possible flights available, the probability of at least one flight being delayed is $\frac{1}{5}$. Have students plan, describe in words, and conduct a simulation to find the experimental probability that two flights selected at random will not be delayed.

Suppose you select a number at random from the sample space {1, 2, 3, 4, 5, 6, 7, 8, 9}. Find each theoretical probability.

34. P(the number is a multiple of 3) $\frac{1}{3}$

35. P(the number is less than 5) $\frac{4}{9}$

36. P(the number is prime) $\frac{4}{9}$

37. P(the number is even) $\frac{4}{9}$

38. Suppose you roll two number cubes.

a. What is the sample space?

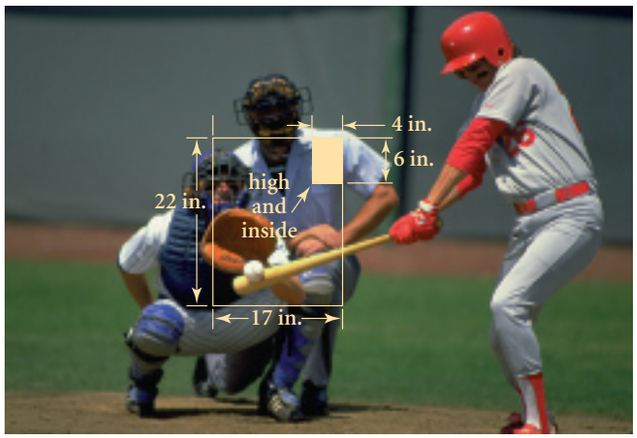
38a. (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

b. How many outcomes are there? **36 outcomes**

c. What is the theoretical probability of getting a sum of 12? $\frac{1}{36}$

d. What is the theoretical probability of getting a sum of 7? $\frac{1}{6}$

39. Sports The batter's strike zone depends on the height and stance of the batter. Find the geometric probability that a baseball thrown at random within the batter's strike zone as shown in the figure below will be a high-inside strike. This is one of the hardest pitches to hit! **≈6.4%**



GO Online Homework Help

Visit: PHSchool.com
Web Code: age-0106

40. a. Sports Team A has won one game and team B has won three games in a World Series. What is the experimental probability that team A wins the next game? That team B wins the next game? $\frac{1}{4}, \frac{3}{4}$

b. Critical Thinking Do you think that experimental probability is a good predictor of the winner of the next game? Explain. **Answers may vary. Sample: Variables such as injuries make probability a poor predictor.**

41. Writing Explain what you would need to know to determine the theoretical probability that a five-digit postal ZIP code ends in 1. **if there are any restrictions on the last digit of a ZIP code**

42. Suppose you choose a two-digit number at random. What is the theoretical probability that its square root is an integer? $\frac{1}{15}$, **or 6.7%**

Challenge

43. Assume that an event is neither certain nor impossible. Then the odds in favor of the event are the ratio of the number of favorable outcomes to the number of unfavorable outcomes. **See left.**

43a. $\frac{a}{a+b}$

a. If the odds in favor of the event are a to b or $\frac{a}{b}$, what is the probability of the event?

b. a to $b - a$ or $\frac{a}{b-a}$

b. If the probability of the event is $\frac{a}{b}$, what are the odds in favor of the event?

c. A game where the probability of winning is $\frac{1}{2}$; when the odds of winning are $\frac{1}{2}$, the probability of winning is only $\frac{1}{3}$.

c. Would you rather play a game where your odds of winning are $\frac{1}{2}$, or a game where your probability of winning is $\frac{1}{2}$? Explain.

44. Open-Ended Use a telephone book. Select 50 telephone numbers at random and record the first three digits (the "exchange") of each number. Summarize your results using probability statements. **Check students' work.**

45. On a TV game show, you want to win a prize that is hidden behind one of three doors. You choose one door, but before it is opened the host opens another door and shows that the prize is not there. Now you can switch to the other unopened door or stick with your original choice.

a. Find the experimental probability of winning the prize if you stick with your original choice. (*Hint:* Simulate the doors with index cards and the prize with a mark on one side of one card. One person can act as the host and another as the contestant.) about $\frac{1}{3}$

b. Find the experimental probability of winning if you switch to the other door. about $\frac{2}{3}$

Test Prep

Multiple Choice

46. What is the theoretical probability of getting a 2 or a 3 when rolling a number cube? B

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{6}$

47. How many outcomes are in the sample space for rolling a number cube and tossing a coin? H

F. 2 G. 6 H. 12 J. 24

48. The random number table simulates an experiment where you toss a coin 90 times. Even digits represent heads and odd digits represent tails. What is the experimental probability, to the nearest percent, of the coin coming up heads? C

A. 45% B. 50%
C. 54% D. 56%

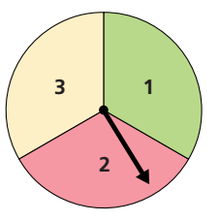
Random Number Table		
31504	51648	40613
79321	80927	42404
15594	84675	68591
34178	00460	31754
49676	58733	00884
85400	72294	22551

Short Response

49–51. See margin.

49. What is the sample space for spinning the spinner at the right twice? Are all the outcomes equally likely?

50. For the spinner at the right, what is the probability of spinning a 1 on both spins? Explain.



Extended Response

51. For the spinner at the right, which is more likely on two spins, an even sum or a sum that is not prime? Include all the steps of your solution.

Mixed Review

Lesson 1-5

Solve each absolute value equation. Check your answers.

52. $|x + 3| = 9$ $-12, 6$

53. $|3x - 5| = 10$ $-\frac{5}{3}, 5$

54. $|2x + 7| + 3 = 22$ $-13, 6$

55. $|3x - 6| - 7 = 14$ $-5, 9$

56. $|2x + 3| - 9 = 14$ $-13, 10$

57. $|6 - 5x| = 18$ $-\frac{12}{5}, \frac{24}{5}$

Lesson 1-5

Solve each absolute value inequality.

58. $2|x| - 3 \geq 5$ $x \leq -4$ or $x \geq 4$

59. $|2x - 4| + 16 \leq 24$ $-2 \leq x \leq 6$

60. $|3x - 5| - 2 > 0$ $x < 1$ or $x > 2\frac{1}{3}$

61. $|2x + 4| - 6 < 0$ $-5 < x < 1$

62. $2|x + 3| \geq 10$ $x \leq -8$ or $x \geq 2$

63. $6|x + 9| \leq 36$ $-15 \leq x \leq -3$

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 51
- Test-Taking Strategies, p. 46
- Test-Taking Strategies with Transparencies

51. [4] a. The favorable outcomes for an even sum are 11, 13, 22, 31, and 33.

b. The probability of an even sum is $\frac{\text{no. of favorable outcomes}}{\text{total no. of outcomes}} = \frac{5}{9}$.

c. The favorable outcomes for a composite sum are 13, 22, 31, and 33.

d. The probability of a composite sum is $\frac{4}{9}$.

e. $\frac{5}{9} > \frac{4}{9}$, so an even sum is more likely than a composite sum.

[3] omits one of the five parts of the answer

[2] omits two OR three of the five parts of the answer

[1] omits four of the five parts of the answer

pages 42–45 Exercises

49. [2] (11, 12, 13, 21, 22, 23, 31, 32, 33); all the pairs are equally likely because the three regions have the same area (or are congruent), and the outcome of the first spin does not affect the outcome of the second spin.

[1] answers one of the two parts

50. [2] $\frac{1}{9}$; the probability of spinning a 1 on each spin is $\frac{1}{3}$. The probability of spinning a 1 on both spins is $\frac{1}{3} \cdot \frac{1}{3}$ or $\frac{1}{9}$.

[1] no explanation included