3-4 Linear Programming

What You'll Learn

- To find maximum and minimum values
- To solve problems with linear programming

. . . And Why

To maximize profit, as in Example 2

Check Skills You'll Need

GO for Help Lessons 3-2 and 3-3

Solve each system of equations.

1. $\begin{cases} y = -3x + 3 \\ y = 2x - 7 \end{cases}$ **(2, −3)**
2. $\begin{cases} x + 2y = 5 \\ x - y = -1 \end{cases}$ **(1, 2)**
3. $\begin{cases} 4x + 3y = 7 \\ 2x - 5y = -3 \end{cases}$ **(1, 1)**

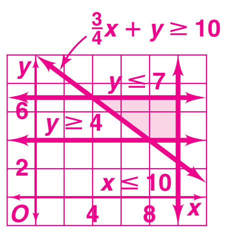
Solve each system of inequalities by graphing. **4–6. See back of book.**

4. $\begin{cases} x \geq 5 \\ y > -3x + 6 \end{cases}$
5. $\begin{cases} 3y > 5x + 2 \\ y \leq -x + 7 \end{cases}$
6. $\begin{cases} x + 3y < -6 \\ 2x - 3y \leq 4 \end{cases}$

New Vocabulary
- linear programming
- objective function
- constraints
- feasible region

1 Finding Maximum and Minimum Values

2.



Activity: Finding a Minimum Value

Music Suppose you want to buy some tapes and CDs. You can afford as many as 10 tapes or 7 CDs. You want at least 4 CDs and at least 10 hours of recorded music. Each tape holds about 45 minutes of music, and each CD holds about an hour.

1. Write a system of inequalities to model the problem. Let x represent the number of tapes purchased. Let y represent the number of CDs purchased. $\begin{cases} x \leq 10 \\ 4 \leq y \leq 7 \\ \frac{3x}{4} + y \geq 10 \end{cases}$
2. Graph your system of inequalities. **See left.**
3. Does each ordered pair satisfy the system you have graphed?
   a. (4, 7) **yes** b. (12, 7) **no** c. (7, 6) **yes** d. (9, 4) **yes** e. (10, 4) **yes**

Linear programming is a technique that identifies the minimum or maximum value of some quantity. This quantity is modeled with an **objective function.** Limits on the variables in the objective function are **constraints,** written as linear inequalities.

Vocabulary Tip

Constraints are sometimes referred to as restrictions.

The first paragraph of the Investigation describes the constraints on buying tapes and CDs. Suppose you buy x tapes and y CDs. The constraints on x and y can be modeled with inequalities as follows.

as many as 10 tapes	$x \leq 10$	at least 4 CDs	$y \geq 4$
as many as 7 CDs	$y \leq 7$	at least 10 hours	$\frac{3}{4}x + y \geq 10$

Differentiated Instruction Solutions for All Learners

Special Needs L1
After students have graphed the restrictions, ask them to label their graph with each of the following: *left boundary, right boundary, upper boundary,* and *lower boundary.* Then discuss why the maximum and minimum values occur at the vertices.
learning style: visual

Below Level L2
Linear programming can be quite abstract for some students. Carefully review all the vocabulary terms and new concepts.
learning style: verbal

3-4

1. Plan

Objectives
1 To find maximum and minimum values
2 To solve problems with linear programming

Examples
1 Testing Vertices
2 Real-World Connection

Math Background

Professional Development

Linear programming is an extension of solving linear inequalities. You are given constraints represented by linear inequalities that are graphed. All of the points in the overlapping region are solutions, but linear programming problems are usually looking for maximum or minimum values. You substitute the coordinates of the vertices into an objective function to determine which yield the maximum or minimum value.

More Math Background: p. 116D

Lesson Planning and Resources

See p. 116E for a list of the resources that support this lesson.

Bell Ringer Practice

PowerPoint

Check Skills You'll Need
For intervention, direct students to:

Graphing Systems of Equations
Lesson 3-1: Example 1
Extra Skills and Word Problems Practice, Ch. 3

Solving Systems Algebraically
Lesson 3-2: Examples 1, 3
Extra Skills and Word Problems Practice, Ch. 3

Systems of Inequalities
Lesson 3-3: Example 1
Extra Skills and Word Problems Practice, Ch. 3

2. Teach

Guided Instruction

Activity
The graph of the system of inequalities encloses a region in the coordinate plane. The only points of this region that make sense for this situation are the points that have whole number coordinates.

1 EXAMPLE Visual Learners

After students have graphed the restrictions, have them graph the lines $3x + 2y = 18$, $3x + 2y = 6$, and $3x + 2y = 0$. Discuss with the students why the maximum and minimum values occur at the vertices.

Additional Examples

1 Find the values of x and y that maximize and minimize P if $P = -5x + 4y$.

$$\begin{cases} y \geq -\frac{2}{3}x + \frac{11}{3} \\ y \leq \frac{1}{4}x + \frac{11}{4} \\ y \geq 3x - 11 \end{cases}$$

maximum of 7 when $x = 1$ and $y = 3$, minimum of -16 when $x = 4$ and $y = 1$

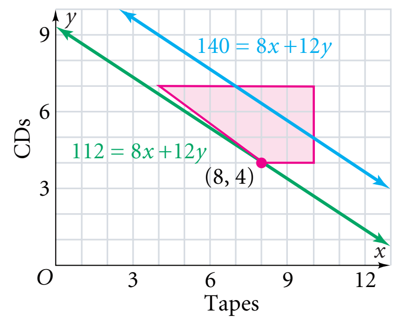
Vocabulary Tip

Feasible means "doable" or "suitable."

The constraints form the system of inequalities at the right. The red region in the graph, the **feasible region,** contains all the points that satisfy all the constraints.

$$\begin{cases} x \leq 10 \\ y \leq 7 \\ y \geq 4 \\ \frac{3}{4}x + y \geq 10 \end{cases}$$

If you buy tapes at \$8 each and CDs at \$12 each, then the objective function for the total cost C is $C = 8x + 12y$. The blue line is the graph for the total cost \$140. The green line is for the total cost \$112.



Graphs of the objective function for various values of C are parallel lines. Lines closer to the origin represent lower costs. The graph closest to the origin that intersects the feasible region intersects it at the vertex (8, 4). The graph of the objective function farthest from the origin that intersects the feasible region intersects it at the vertex (10, 7). Graphs of an objective function that represent a maximum or minimum value intersect a feasible region at a vertex.

Key Concepts

Property **Vertex Principle of Linear Programming**

If there is a maximum or a minimum value of the linear objective function, it occurs at one or more vertices of the feasible region.

1 EXAMPLE Testing Vertices

Multiple Choice What values of x and y maximize P for the objective function $P = 3x + 2y$?

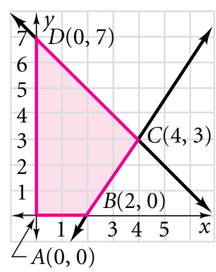
Constraints $\begin{cases} y \geq \frac{3}{2}x - 3 \\ y \leq -x + 7 \\ x \geq 0, y \geq 0 \end{cases}$

(A) (2, 0) (B) (2, 6)
(C) (4, 3) (D) (4, 4)

Test-Taking Tip

Remember to check the value of the objective function at *each* vertex when solving linear programming problems.

Step 1
Graph the constraints.



Step 2
Find coordinates for each vertex.

Vertex
A(0, 0)
B(2, 0)
C(4, 3)
D(0, 7)

Step 3
Evaluate P at each vertex.

$\mathbf{P = 3x + 2y}$
$P = 3(0) + 2(0) = 0$
$P = 3(2) + 2(0) = 6$
$P = 3(4) + 2(3) = 18$
$P = 3(0) + 2(7) = 14$

When $x = 4$ and $y = 3$, P has its maximum value of 18. The answer is C.

Quick Check

1 Use the constraints in Example 1 with the objective function $P = 2x + 3y$. Find the values of x and y that maximize and minimize P. Find the value of P at each point. **See left.**

1. $P = 2(0) + 3(0) = 0$
$P = 2(2) + 3(0) = 4$
$P = 2(4) + 3(3) = 17$
$P = 2(0) + 3(7) = 21$
When $x = 0$ and $y = 7$, P is maximized at 21.
When $x = 0$ and $y = 0$, P is minimized at 0.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Have students use catalogs or the Internet to find products and prices to write a linear programming problem. Have them share the problem and its solution.

learning style: verbal

English Language Learners ELL
Students can easily confuse the new vocabulary terms. Have students write the terms *objective function, constraints,* and *feasible region* next to the corresponding function, system of inequalities, and bounded region on the graph.

learning style: visual

2 Writing Linear Programs

You can use linear programming to solve many real-world problems.

2 EXAMPLE **Real-World Connection**

Profit Suppose you are selling cases of mixed nuts and roasted peanuts. You can order no more than a total of 500 cans and packages and spend no more than \$600. How can you maximize your profit? How much is the maximum profit?

Mixed Nuts
12 cans per case
You pay ......\$24 per case
Sell at ....\$3.50 per can
\$18 profit per case!

Roasted Peanuts
20 packages per case
You pay ......\$15 per case
Sell at ...\$1.50 per package
\$15 profit per case!

Define Let x = number of cases of mixed nuts ordered.
Let y = number of cases of roasted peanuts ordered.
Let P = total profit.

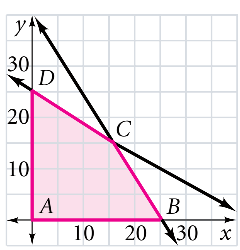
Relate Organize the information in a table.

	Mixed Nuts	Roasted Peanuts	Total	
Number of Cases	x	y	$x+y$	
Number of Units	$12x$	$20y$	500	constraint
Cost	$24x$	$15y$	600	constraint
Profit	$18x$	$15y$	$18x+15y$	objective

Write Write and simplify the constraints. Write the objective function.

$$\begin{cases} 12x + 20y \le 500 \\ 24x + 15y \le 600 \\ x \ge 0,\ y \ge 0 \end{cases} \Rightarrow \begin{cases} 3x + 5y \le 125 \\ 8x + 5y \le 200 \\ x \ge 0,\ y \ge 0 \end{cases} \qquad P = 18x + 15y$$

Step 1
Graph the constraints.



Step 2
Find the coordinates of each vertex.

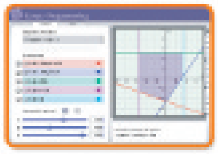
Step 3
Evaluate P at each vertex.

Vertex	$P = 18x + 15y$
$A(0, 0)$	$P = 18(0) + 15(0) = 0$
$B(25, 0)$	$P = 18(25) + 15(0) = 450$
$C(15, 16)$	$P = 18(15) + 15(16) = 510$
$D(0, 25)$	$P = 18(0) + 15(25) = 375$

You can maximize your profit by selling 15 cases of mixed nuts and 16 cases of roasted peanuts. The maximum profit is \$510.

Quick Check 2 If you sell mixed nuts for \$4.25 per can, what should you order to maximize profit? **Order 25 cases of mixed nuts and no cases of roasted peanuts.**

Online active math



For: Linear Program Activity
Use: Interactive Textbook, 3-4

Vocabulary Tip

Linear programming got its name because it provides a plan, or a program, that uses linear equations and inequalities.

Guided Instruction

2 EXAMPLE **Math Tip**

You may wish to point out that the vertices of the feasible region have whole number coordinates and make sense in terms of this situation. If the vertices were not whole numbers, a more detailed analysis would be necessary.

Additional Examples

2 A furniture manufacturer can make from 30 to 60 tables a day and from 40 to 100 chairs a day. It can make at most 120 units in one day. The profit on a table is \$150, and the profit on a chair is \$65. How many tables and chairs should they make per day to maximize profit? How much is the maximum profit? **60 tables, 60 chairs; \$12,900**

Resources
- Daily Notetaking Guide 3-4 L3
- Daily Notetaking Guide 3-4—Adapted Instruction L1

Closure

In a linear programming problem, what values do you test to find the values that maximize or minimize the objective function? **the coordinates of each vertex of the feasible region**

3. Practice

Assignment Guide

1 A B 1-9, 12, 14-19

2 A B 10-11, 13, 20

C Challenge 21-23

Test Prep 24-27

Mixed Review 28-42

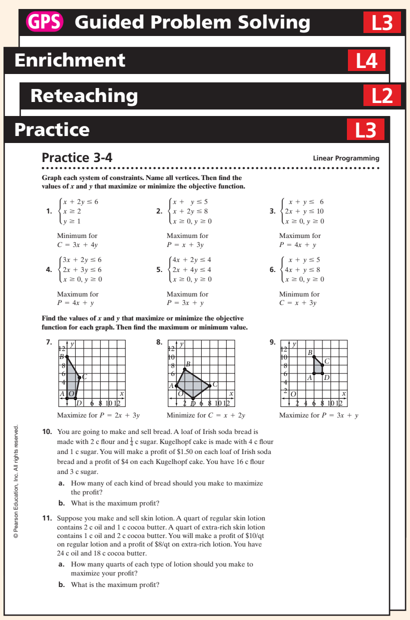
Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 7, 10, 12, 17, 20.

Error Prevention!

Exercises 1–9 Students may think that the "highest" point on a graph is the maximum. Remind students that the only way to determine a maximum or a minimum is through substitution of the vertices' coordinates into the objective function to find the greatest and least values of the objective function.

Differentiated Instruction Resources



EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 140)

GO for Help

Find the values of x and y that maximize or minimize the objective function for each graph. **1–3. See left.**

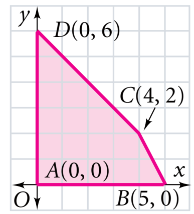
1. When $x = 4$ and $y = 2$, P is maximized at 16.

2. When $x = 600$ and $y = 0$, P is maximized at 4200.

3. When $x = 6$ and $y = 8$, C is minimized at 36.

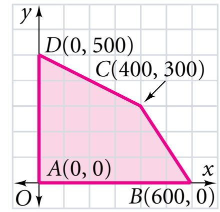
4–9. See back of book.

1.



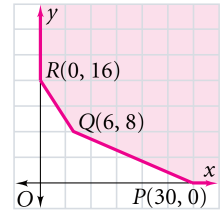
Maximum for $P = 3x + 2y$

2.



Maximum for $P = 7x + 4y$

3.



Minimum for $C = 2x + 3y$

Graph each system of constraints. Name all vertices. Then find the values of x and y that maximize or minimize the objective function.

4. $\begin{cases} x \le 5 \\ y \le 4 \\ x \ge 0, y \ge 0 \end{cases}$ Maximum for $P = 3x + 2y$

5. $\begin{cases} x + y \ge 8 \\ y \ge 5 \\ x \ge 0 \end{cases}$ Minimum for $P = 3x + 2y$

6. $\begin{cases} x + y \le 8 \\ 2x + y \le 10 \\ x \ge 0, y \ge 0 \end{cases}$ Maximum for $N = 100x + 40y$

7. $\begin{cases} x + y \ge 6 \\ x \le 8 \\ y \le 5 \end{cases}$ Minimum for $C = x + 3y$

8. $\begin{cases} x + 2y \ge 8 \\ x \ge 2 \\ y \ge 0 \end{cases}$ Minimum for $C = x + 3y$

9. $\begin{cases} 2 \le x \le 6 \\ 1 \le y \le 5 \\ x + y \le 8 \end{cases}$ Maximum for $P = 3x + 2y$

Example 2 (page 141)

10. Ecology Teams chosen from 30 forest rangers and 16 trainees are planting trees. An experienced team consisting of two rangers can plant 500 trees per week. A training team consisting of one ranger and two trainees can plant 200 trees per week.

	Experienced Teams	Training Teams	Total
Number of Teams	x	y	$x + y$
Number of Rangers	$2x$	y	30
Number of Trainees	0	$2y$	16
Number of Trees Planted	$500x$	$200y$	$500x + 200y$

10a. $\begin{cases} 2x + y \le 30 \\ 2y \le 16 \\ x \ge 0, y \ge 0 \end{cases}$ $P = 500x + 200y$

a. Write an objective function and constraints for a linear program that models the problem.

b. How many of each type of team should be formed to maximize the number of trees planted? How many trainees are used in this solution? How many trees are planted? **15 experienced teams, 0 training teams; none; 7500**

c. Find a solution that uses all the trainees. How many trees will be planted in this case? **11 experienced teams; 8 training teams; 7100 trees**

11. Air Quality Trees in urban areas help keep air fresh by absorbing carbon dioxide. A city has $2100 to spend on planting spruce and maple trees. The land available for planting is 45,000 ft². How many of each tree should the city plant to maximize carbon dioxide absorption? **70 spruce; 0 maple**

Facts for a Single Tree

	Spruce	Maple
Planting Cost	$30	$40
Area Required	600 ft²	900 ft²
Carbon Dioxide Absorption	650 lb/yr	300 lb/yr

SOURCES: Auburn University and Anderson & Associates

B Apply Your Skills

12. Writing Explain why solving a system of linear equations is a necessary skill for linear programming. **Solving a system of linear equations is a necessary skill used to locate the vertex points.**

Real-World Connection

Careers A microbiologist studies microorganisms, such as bacteria and viruses, to determine their structure and function.

13. Multiple Choice A biologist is developing two new strains of bacteria. Each sample of Type I bacteria produces four new viable bacteria, and each sample of Type II produces three new viable bacteria. Altogether, at least 240 new viable bacteria must be produced. At least 30, but not more than 60, of the original samples must be Type I. Not more than 70 of the samples can be Type II. A sample of Type I costs $5 and a sample of Type II costs $7. How many samples of Type II bacteria should be used to minimize cost? **A**

(A) 0 (B) 30 (C) 60 (D) 70

Graph each system of constraints. Name all vertices. Then find the values of x and y that maximize or minimize the objective function. Find the maximum or minimum value. **14–19. See back of book.**

14. $\begin{cases} 3x + y \le 7 \\ x + 2y \le 9 \\ x \ge 0, y \ge 0 \end{cases}$ Maximum for $P = 2x + y$

15. $\begin{cases} 25 \le x \le 75 \\ y \le 110 \\ 8x + 6y \ge 720 \end{cases}$ Minimum for $C = 8x + 5y$

16. $\begin{cases} x + y \le 11 \\ 2y \ge x \\ x \ge 0, y \ge 0 \end{cases}$ Maximum for $P = 3x + 2y$

17. $\begin{cases} 2x + y \le 300 \\ x + y \le 200 \\ x \ge 0, y \ge 0 \end{cases}$ Maximum for $P = x + 2y$

18. $\begin{cases} 5x + y \ge 10 \\ x + y \ge 6 \\ x + 4y \ge 12 \\ x \ge 0, y \ge 0 \end{cases}$ Minimum for $C = 10{,}000x + 20{,}000y$

19. $\begin{cases} 6 \le x + y \le 13 \\ x \ge 3 \\ y \ge 1 \end{cases}$ Maximum for $P = 4x + 3y$

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20. Cooking Baking a tray of corn muffins takes 4 c milk and 3 c wheat flour. A tray of bran muffins takes 2 c milk and 3 c wheat flour. A baker has 16 c milk and 15 c wheat flour. He makes $3 profit per tray of corn muffins and $2 profit per tray of bran muffins. How many trays of each type of muffin should the baker make to maximize his profit? **3 trays of corn muffins and 2 trays of bran muffins**

C Challenge

21. A vertex of a feasible region does not always have whole-number coordinates. Sometimes you may need to round coordinates to find the solution. Using the objective function and the constraints at the right, find the whole-number values of x and y that minimize C. Then find C for those values of x and y. **See back of book.**

$C = 6x + 9y$

$\begin{cases} x + 2y \ge 50 \\ 2x + y \ge 60 \\ x \ge 0, y \ge 0 \end{cases}$

22. Open-Ended Write a system of constraints whose graphs determine a trapezoid. Write an objective function and evaluate it at each vertex. **Check students' work.**

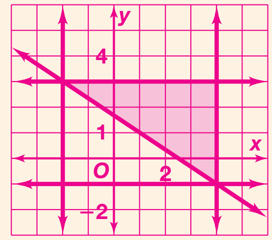
4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Graph the system of constraints. Name all vertices of the feasible region. Then find the values of x and y that maximize and minimize the objective function $P = 2x + 7y + 4$.

$\begin{cases} -2 \le x \le 4 \\ -1 \le y \le 3 \\ y \ge -\frac{2}{3}x + \frac{5}{3} \end{cases}$



(−2, 3), (4, 3), (4, −1); maximum of 33 when $x = 4$ and $y = 3$, minimum of 5 when $x = 4$ and $y = -1$

2. If the constraint on y in the system for Question 1 is changed to $1 \le y \le 3$, how does the minimum value for the objective function change? **There is a new minimum value of 13 when $x = 1$ and $y = 1$.**

Alternative Assessment

Have students work in small groups. Each group should write and solve a real-world linear programming problem. Groups then exchange problems and solve. Groups should compare answers and resolve any differences.

Test Prep

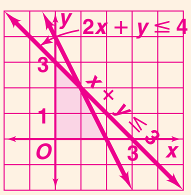
Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies

pages 142–144 Exercises

26. [2] The boundary line through $R(0, 40)$ and $Q(10, 20)$ is $y = -2x + 40$, so the constraint is $y \geq -2x + 40$. The boundary line through $Q(10, 20)$ and $P(50, 0)$ is $y = -\frac{1}{2}x + 25$, so the constraint is $y \geq -\frac{1}{2}x + 25$.

[1] includes only one of the two parts of the answer above OR makes a minor error in calculation

27. [4]



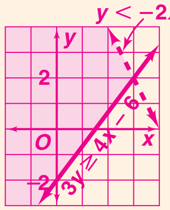
The vertices are (0, 0), (2, 0), (0, 3), and (1, 2).

[3] incorrectly graphs equations, but interprets inequalities correctly

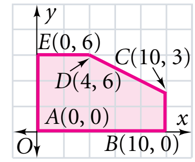
[2] answer of vertices only

[1] only 2 correct vertices with no work shown

28.



23. **Critical Thinking** Sometimes two corners of a graph both yield the maximum profit. In this case, many other points may also yield the maximum profit. Evaluate the profit formula $P = x + 2y$ for the graph shown. Find four points that yield the maximum profit.
Answers may vary. Sample: (4, 6), (6, 5), (9, 3.5), (10, 3)



Test Prep

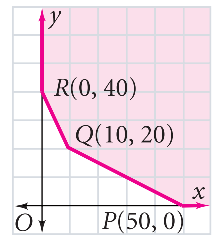
Multiple Choice

24. Which point maximizes $N = 4x + 3y$ and lies within the feasible region of the constraints at the right? **C**
$\begin{cases} y \leq 9 \\ 2x + 2y \leq 18 \\ x \leq 3 \end{cases}$
A. (0, 0) B. (9, 0) C. (3, 6) D. (0, 9)

25. The vertices of a feasible region are (0, 0), (0, 2), (5, 2), and (4, 0). For which objective function is the maximum cost C found at the vertex (4, 0)? **H**
F. $C = -2x + 3y$
G. $C = 2x + 7y$
H. $C = 4x - 3y$
J. $C = 5x + 3y$

Short Response

26. The figure at the right shows the feasible region for a system of constraints. This system includes $x \geq 0$ and $y \geq 0$. Find the remaining constraint(s). **See margin.**



Extended Response

27. What are the vertices of the feasible region bounded by the constraints at the right? **See margin.**
$\begin{cases} x + y \leq 3 \\ 2x + y \leq 4 \\ x \geq 0, y \geq 0 \end{cases}$

Mixed Review

Go for Help

Lesson 3-3 **Solve each system of inequalities by graphing.**

28. $\begin{cases} y < -2x + 8 \\ 3y \geq 4x - 6 \end{cases}$ **29.** $\begin{cases} x - 2y \geq 11 \\ 5x + 4y < 27 \end{cases}$ **30.** $\begin{cases} 2x + 6y > 12 \\ 3x + 9y \leq 27 \end{cases}$

31. $\begin{cases} 2y + x < 4 \\ y - 2x \geq 4 \end{cases}$ **32.** $\begin{cases} y + 5 \geq -2x \\ y - x \geq -2 \end{cases}$ **33.** $\begin{cases} 2y - 4x < 6 \\ 6x < 3y + 12 \end{cases}$

28–33. See margin pp. 144–145.

Lesson 2-4 **34.** **Data Analysis** Use the data below.

A Survey of Paperback Books: How Long and How Much?

Pages	326	450	246	427	208	339	367	445	404	465	378	265
Price ($)	7.50	7.99	6.99	7.99	6.99	7.95	7.50	7.95	7.95	7.99	7.99	6.99

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For: Graphing calculator procedures
Web Code: age-2109

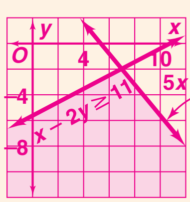
a. Make a scatter plot of the data. **See back of book.**
b. What kind of correlation do you see? **positive**
c. Find a linear model. **c–d. See margin p. 145.**
d. What price would you predict for a paperback containing 100 pages?

Lesson 1-2 **Evaluate each expression for $a = 3$ and $b = -5$.**

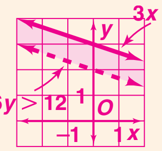
35. $2a + b$ **1** **36.** $a - b$ **8** **37.** $-4 + 2ab$ **−34** **38.** $a + \frac{3b}{a}$ **−2**

39. $3(a - b)$ **24** **40.** $4a - 2 + 3b$ **−5** **41.** $\frac{a - b}{2a}$ $\frac{4}{3}$ **42.** $b(2b - a)$ **65**

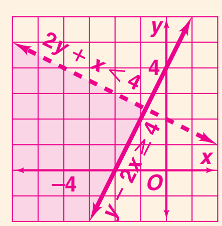
29.



30.



31.



3-5

1. Plan

Objectives
1 To graph points in three dimensions
2 To graph equations in three dimensions

Examples
1 Graphing in Coordinate Space
2 Real-World Connection
3 Sketching a Plane

Professional Development

Math Background

Since space is three-dimensional, a three-dimensional coordinate system describes positions in space. The points in this system have three coordinates, usually *x, y,* and *z.* To locate a point in a three-dimensional coordinate system, you start at the origin and first move forward or backward, then right or left, and finally up or down.

More Math Background: p. 116D

Lesson Planning and Resources

See p. 116E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:
Linear Equations
Lesson 2-2: Examples 1, 2
Extra Skills and Word Problems Practice, Ch. 2

3-5 Graphs in Three Dimensions

What You'll Learn
- To graph points in three dimensions
- To graph equations in three dimensions

. . . And Why
To locate points on a virtual bicycle helmet, as in Example 2

Check Skills You'll Need — GO for Help Lesson 2-2

Find the x- and y-intercepts of the graph of each linear equation.

1. $y = 2x + 6$ **x: –3, y: 6**
2. $2x + 9y = 36$ **x: 18, y: 4**
3. $3x - 8y = -24$ **x: –8, y: 3**
4. $4x - 5y = 40$ **x: 10, y: –8**

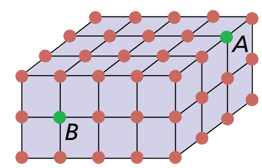
Graph each linear equation. **5–8. See back of book.**

5. $y = 3x$
6. $y = -2x + 4$
7. $4y = 3x - 8$
8. $-3x - 2y = 7$

New Vocabulary • coordinate space • ordered triples • trace

1 Graphing Points in Three Dimensions

Suppose you want to describe how to get from point A to point B along the grid shown at the right. You could say "Move down one unit, forward two units, and left three units," or "Move left three units, forward two units, and down one unit."

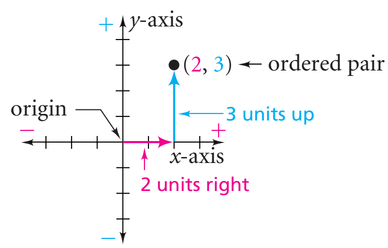


To describe positions in space, you need a three-dimensional coordinate system.

You have learned to graph on an xy-coordinate plane using ordered pairs. Adding a third axis, the z-axis, to the xy-coordinate plane creates **coordinate space.** In coordinate space you graph points using **ordered triples** of the form (x, y, z).

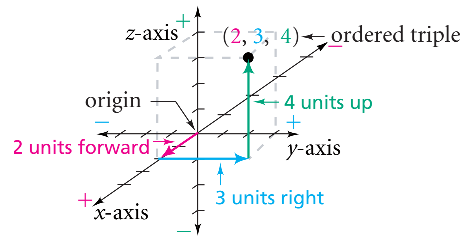
Real-World Connection
The global positioning system locates persons or objects in three dimensions.

Points in a Plane



A two-dimensional coordinate system allows you to graph points in a plane.

Points in Space



A three-dimensional coordinate system allows you to graph points in space.

In the coordinate plane, point (2, 3) is two units right and three units up from the origin. In coordinate space, point (2, 3, 4) is two units forward, three units right, and four units up.

Differentiated Instruction Solutions for All Learners

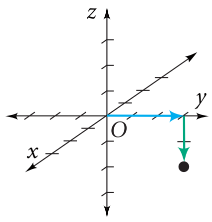
Special Needs L1
Make rectangular prisms from small boxes by dividing each face into a square grid as shown in the text. Have students label points *A* and *B* on different faces and illustrate with colors how to move along the grid to get from one point to the other.
learning style: tactile

Below Level L2
To assist students in visualizing graphs in three dimensions, set up a coordinate system in your classroom. Use tape and string to represent graphs.
learning style: visual

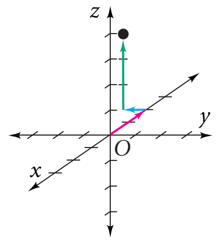
1 EXAMPLE Graphing in Coordinate Space

Graph each point in coordinate space.

a. $(0, 3, -2)$
Sketch the axes. From the origin, move right 3 units and down 2 units.



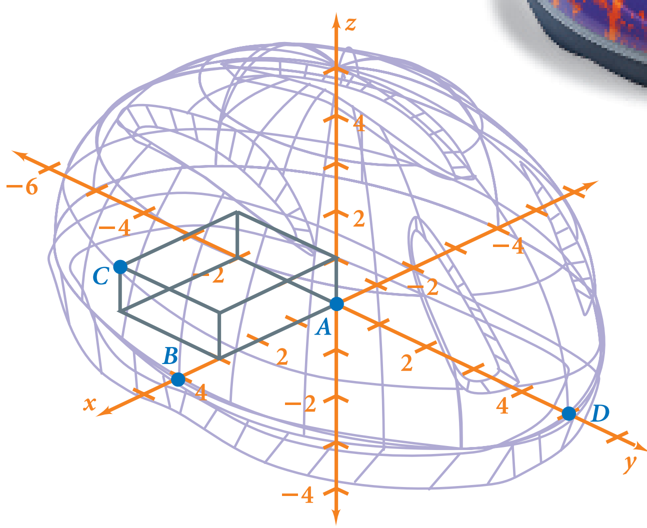
b. $(-2, -1, 3)$
Sketch the axes. From the origin, move back 2 units, left 1 unit, and up 3 units.



Quick Check 1 Graph each point in coordinate space. **a–d. See back of book.**
a. $(0, -4, -2)$ **b.** $(-1, 1, 3)$ **c.** $(3, -5, 2)$ **d.** $(3, 3, -3)$

2 EXAMPLE Real-World Connection

Product Design Computers are used to design three-dimensional objects. Programs allow the designer to view the object from different perspectives. Find coordinates for points A, B, and C in the diagram below.



$A(0, 0, 0)$, $B(4, 0, 0)$, $C(3, -2, 1)$

Quick Check 2 **a.** Find coordinates for point D in the diagram. **(0, 5, 0)**
b. Does the point $(-3, 2, 4)$ lie inside or outside the helmet? Explain.
outside the helmet, since the helmet is curved

2. Teach

Guided Instruction

1 EXAMPLE Connection to Geometry

Ask students to name three dimensions used to describe a prism. **length, width, and height**

2 EXAMPLE Careers

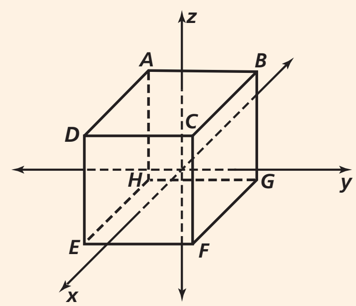
Product designers' diagrams have the viewer situated in the part of space where all three coordinates are positive. Ask students why it is important for a designer to be able to view an object from several different perspectives.

PowerPoint

Additional Examples

1 Graph each point in coordinate space.
a. $(-3, 3, -4)$ **b.** $(-3, -4, 2)$
See back of book.

2 In the diagram, the origin is at the center of a cube that has edges 6 units long. The x-, y-, and z-axes are perpendicular to the faces of the cube. Give the coordinates of the corners of the cube.



***A*(−3, −3, 3), *B*(−3, 3, 3), *C*(3, 3, 3), *D*(3, −3, 3), *E*(3, −3, −3), *F*(3, 3, −3), *G*(−3, 3, −3), *H*(−3, −3, −3)**

Advanced Learners L4
Have students research how computer graphics imaging works and share their findings with the class.
learning style: verbal

English Language Learners ELL
Have students look up the word *trace* in a dictionary. Have them compare the English meanings to the mathematical definition, a line formed by the intersection of a plane with the *xy*-plane, the *yz*-plane, or the *zx*-plane.
learning style: verbal

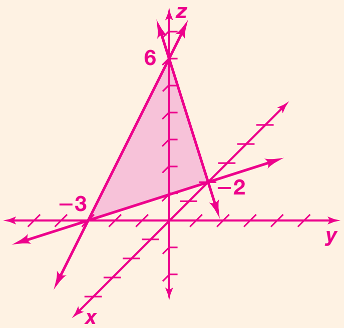
Guided Instruction

3 EXAMPLE Math Tip

Stress similarities with the corresponding concepts for two dimensions. In two dimensions, the intercepts of a line are the points where the line intersects the *x*- and *y*-axes. In three dimensions, the intercepts of a plane are the points where the plane intersects the *x*-, *y*-, and *z*-axes. The traces associated with a linear equation $Ax + By + Cz = D$ are the lines that are the intersections of the graph of the equation with the *xy*-, *xz*-, and *yz*-planes.

Additional Examples

3 Sketch the graph of $-3x - 2y + z = 6$.



Resources

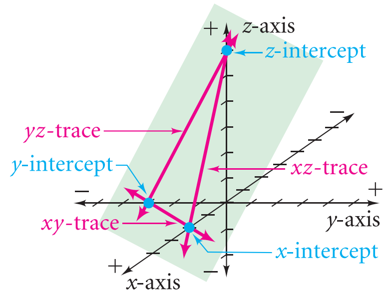
- Daily Notetaking Guide 3-5 L3
- Daily Notetaking Guide 3-5—Adapted Instruction L1

Closure

Describe how you would locate the point $(-2, 4, -3)$ in a three-dimensional coordinate system. **Start at the origin. Move 2 units in the negative direction along the *x*-axis. Then move parallel to the *y*-axis 4 units in the positive direction. Then move parallel to the *z*-axis 3 units in the negative direction.**

2 Graphing Equations in Three Dimensions

The graph of an equation is a picture of all the solutions to the equation. In two dimensions, the graph of $3x - 2y = 6$ is a line. In three dimensions, the graph of $3x - 2y + z = 6$ is a plane, as shown at the right.



Vocabulary Tip

The x-intercept of a line or plane is the point of intersection with the *x*-axis.

If the graph of a plane intersects one of the coordinate planes in a line, then the line is called a **trace.** For example, the *xy*-trace is the line in the *xy*-plane that contains the *x*- and the *y*-intercepts. The *xz*-trace and the *yz*-trace are defined similarly.

To sketch a plane, find the *x*-, *y*-, and *z*- intercepts. Then draw the traces and show the plane with shading.

3 EXAMPLE Sketching a Plane

Sketch the graph of $2x + 3y + 4z = 12$.

Step 1 Find the intercepts.

$$2x + 3y + 4z = 12$$

$$2x + 3(0) + 4(0) = 12$$ **To find the *x*-intercept, substitute 0 for *y* and *z*.**

$$2x = 12$$

$$x = 6$$ **The *x*-intercept is 6.**

$$2(0) + 3y + 4(0) = 12$$ **To find the *y*-intercept, substitute 0 for *x* and *z*.**

$$3y = 12$$

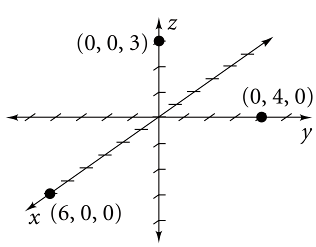
$$y = 4$$ **The *y*-intercept is 4.**

$$2(0) + 3(0) + 4z = 12$$ **To find the *z*-intercept, substitute 0 for *x* and *y*.**

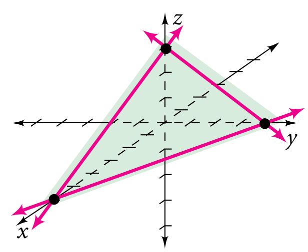
$$4z = 12$$

$$z = 3$$ **The *z*-intercept is 3.**

Step 2 Graph the intercepts.



Step 3 Draw the traces. Shade the plane.



Each point on the plane represents a solution to $2x + 3y + 4z = 12$.

Quick Check

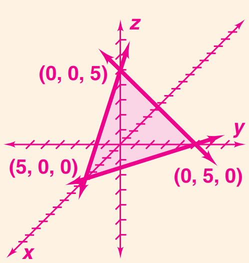
3 Sketch the graph of each equation.

a. $x + y + z = 5$ **b.** $2x - y + 3z = 6$ **c.** $x + 2y - z = -4$

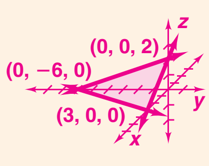
a–c. See margin.

page 148 Quick Check

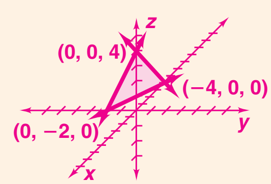
3a.



b.



c.



pages 149–151 Exercises

1. 1 unit back, 5 units right

2. 3 units forward, 3 units left, four units up

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 147)

GO for Help

Describe the location of each point in coordinate space. 1–7. See margin.

1. $(-1, 5, 0)$ **2.** $(3, -3, 4)$

3. $(2, 0, 5)$ **4.** $(-4, -7, -1)$

Graph each point in coordinate space.

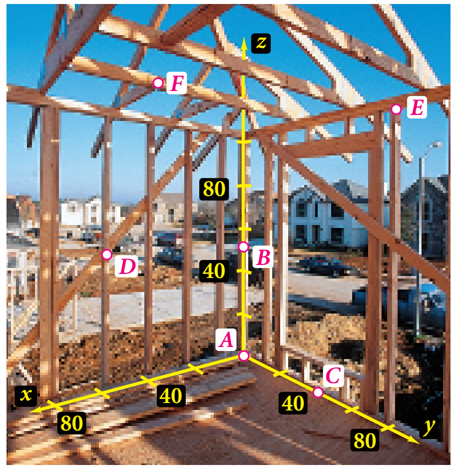
5. $(5, 0, -2)$ **6.** $(0, 0, 4)$

7. $(10, -2, -5)$ **8.** $(-1, -1, -1)$

9. $(-4, -5, 3)$ **10.** $(25, 40, -30)$

11. $(1, 1, 0)$ **12.** $(0, -2, 2)$

8–12. See back of book.



Example 2 (page 147)

Find the coordinates of each point in the diagram.

13. A (0, 0, 0) **14.** B (0, 0, 50) **15.** C (0, 40, 0)

16. D (60, 0, 50) **17.** E (0, 80, 100) **18.** F (60, 30, 100)

Example 3 (page 148)

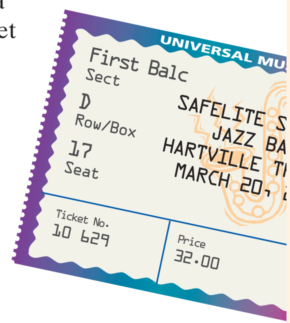
Sketch the graph of each equation. 19–24. See back of book.

19. $x + y + 2z = 4$ **20.** $x + y + z = 2$ **21.** $2x + 6y + z = 6$

22. $x - y - 4z = 8$ **23.** $-x + 3y + z = 6$ **24.** $2x - y - 5z = 10$

B Apply Your Skills

25. **Writing** While visiting friends in New York, you go to a concert. Explain how the seat information on your ticket at the right represents a point in three-dimensional coordinate space. See back of book.



26. **Multiple Choice** Suppose you have \$20 to spend on party decorations. Balloons are \$.05 each, streamers are \$.25 each, and noisemakers are \$.40 each. Which equation best models this situation? C

Ⓐ $5b + 25s + 40n = 20$

Ⓑ $20(b + s + n) = 0.05 + 0.25 + 0.4$

Ⓒ $0.05b + 0.25s + 0.4n = 20$

Ⓓ $0.05b + 0.25s + 0.4n + 20 = 0$

Sketch the graph of each equation. 27–36. See back of book.

27. $7x + 14y - z = 7$ **28.** $-3x + 5y + 10z = 15$

29. $32x + 16y - 8z = 32$ **30.** $-25x + 30y + 50z = 75$

31. $50x + 25y + 100z = 200$ **32.** $14x - 8y + 28z = 28$

GO Online Homework Help
Visit: PHSchool.com
Web Code: age-0305

Sketch the graph of each equation and find the equation of each trace.

GPS **33.** $6x + 6y - 12z = 36$ **34.** $-20x + 10y + 50z = 100$

35. $-12x - 32y - 48z = 96$ **36.** $25x + 125y - 25z = 125$

3. Practice

Assignment Guide

1 A B 1-18, 25, 37-46

2 A B 19-24, 26-36

C Challenge 48-52

Test Prep 53-57

Mixed Review 58-62

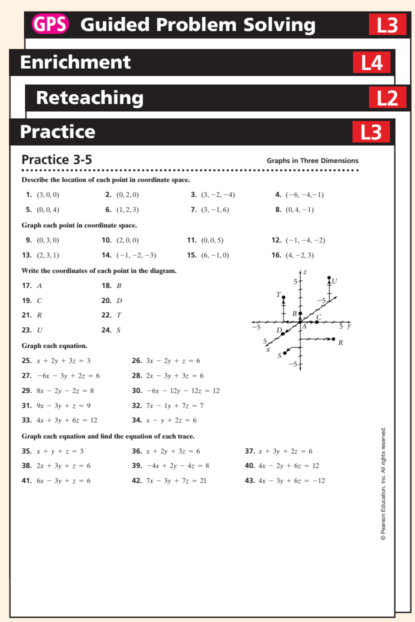
Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 19, 25, 28, 31, 45.

Error Prevention!

Exercises 5–12 Urge students to be careful to refer to the appropriate axis for each coordinate. Diagrams for three-dimensional coordinate systems do not show the x- and y-axes in the positions to which students have grown accustomed from their work in two dimensions.

Differentiated Instruction Resources


GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Practice L3

Practice 3-5 Graphs in Three Dimensions

Describe the location of each point in coordinate space.

1. (3, 0, 0) 2. (0, 2, 0) 3. (3, −2, −4) 4. (−6, −4, −1)
5. (0, 0, 4) 6. (1, 2, 3) 7. (3, −1, 6) 8. (0, 4, −1)

Graph each point in coordinate space.

9. (0, 3, 0) 10. (2, 0, 0) 11. (0, 0, 5) 12. (−1, −4, −2)
13. (2, 3, 1) 14. (−1, −2, −3) 15. (6, −1, 0) 16. (4, −2, 3)

Write the coordinates of each point in the diagram.

17. A 18. B
19. C 20. D
21. R 22. T
23. U 24. S

Graph each equation.

25. $x + 2y + 3z = 3$ 26. $3x - 2y + z = 6$
27. $-6x - 3y + 2z = 6$ 28. $2x - 3y + 3z = 6$
29. $8x - 2y - 2z = 8$ 30. $-6x - 12y - 12z = 12$
31. $9x - 3y + z = 9$ 32. $7x - 1y + 7z = 7$
33. $4x + 3y + 6z = 12$ 34. $x - y + 2z = 6$

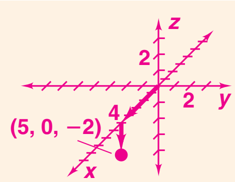
Graph each equation and find the equation of each trace.

35. $x + y + z = 3$ 36. $x + 2y + 3z = 6$ 37. $x + 3y + 2z = 6$
38. $2x + 3y + z = 6$ 39. $-4x + 2y - 4z = 8$ 40. $4x - 2y + 6z = 12$
41. $6x - 3y + z = 6$ 42. $7x - 3y + 7z = 21$ 43. $4x - 3y + 6z = -12$

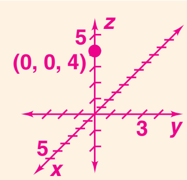
3. 2 units forward, 5 units up

4. 4 units back, 7 units left, 1 unit down

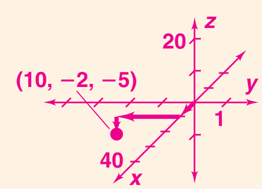
5.



6.



7.



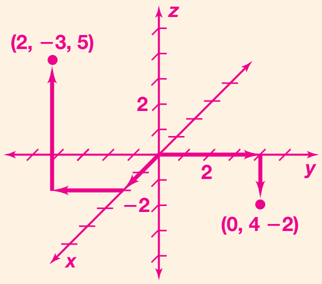
4. Assess & Reteach

PowerPoint

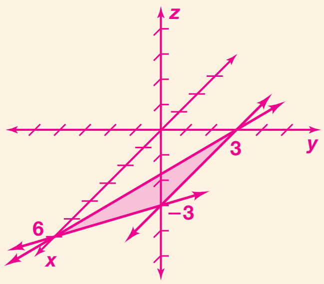
Lesson Quiz

Graph each point in coordinate space.

1. $(2, -3, 5)$ 2. $(0, 4, -2)$



3. Graph $2x + 4y - 4z = 12$.



Alternative Assessment

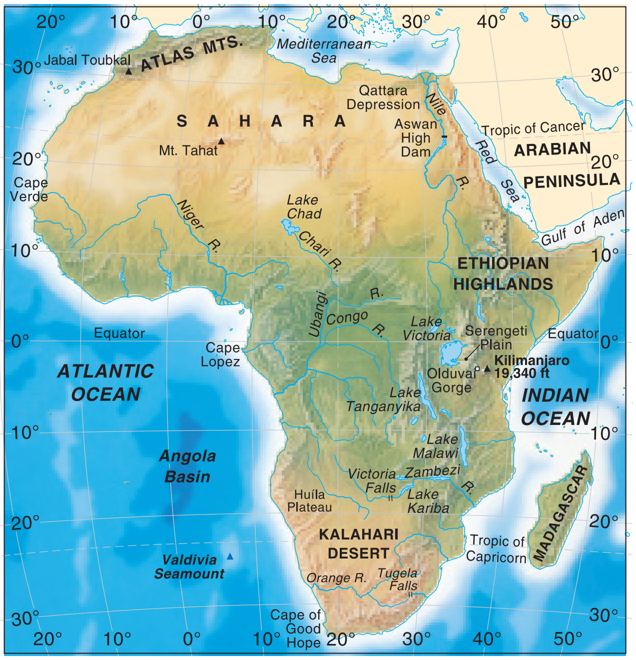
Place a transparency with a three-dimensional coordinate grid on the overhead. Draw a point on the grid and have students describe the location using ordered triples. Repeat a few times. Draw traces representing a plane. Instruct students to find an equation that the plane can represent.

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies



Geography **Use the map to identify the geographic feature found at each location.**

37. latitude 3° S
longitude 37° E
elevation 19,340 ft
Mt. Kilimanjaro

38. latitude 23° N
longitude 5° E
elevation 9,573 ft
Mt. Tahat

39. latitude 25° S
longitude 6° E
elevation −3,072 ft
Valdivia Seamount

40. latitude 15° N
longitude 18° W
elevation 0 ft
Cape Verde

41. latitude 30° N
longitude 27° E
elevation −440 ft
Qattara Depression

42. latitude 13° N
longitude 14° E
elevation 919 ft
Lake Chad

43. latitude 31° N
longitude 8° W
elevation 49,212 ft
Jabal Toubkal

44. latitude 18° S
longitude 25° E
elevation 2,927 ft
Victoria Falls

45. latitude 22° N
longitude 31° E
elevation 600 ft
Aswan High Dam

46. latitude 1° S
longitude 33° E
elevation 3,720 ft
Lake Victoria

47. **Error Analysis** A student claims to find the x-intercept of a plane by substituting 0 for x in the equation of the plane. Explain the student's error. **See margin.**

Challenge

48. **a.** **Geometry** Use the Pythagorean Theorem to find the distance between $(1, 2, 4)$ and $(3, -2, 7)$. (*Hint:* Recall the Distance Formula.) $\sqrt{29}$

b. **Make a Conjecture** Make a conjecture about how to find the coordinates of the midpoint of a segment in coordinate space. **See margin.**

49. **a.** **Critical Thinking** Does every plane have three traces? Explain.

b. Must any two traces of a plane intersect? Explain. **a–b. See margin p. 151.**

Graph each equation in three-dimensional coordinate space.

50. $x = 3$ **51.** $2x + 3y = 6$ **52.** $y = 0$

50–52. See back of book.

Test Prep

Multiple Choice

53. Which point is NOT on the graph of $2x + 3y - z - 12 = 0$? **D**

A. $(6, 0, 0)$ **B.** $(3, 3, 3)$ **C.** $(0, 4, 0)$ **D.** $(1, 1, 7)$

54. Which point is NOT on the plane with equation $-2x - 3y + 5z = 7$? **G**

F. $(1, 2, 3)$ **G.** $(-2, -3, 5)$ **H.** $(-2, 4, 3)$ **J.** $(-4, 2, 1)$

55. What are the intercepts of $-3x + 5y - 2z = 60$? **B**

A. $x = -180, y = 300, z = -120$ **B.** $x = -20, y = 12, z = -30$
C. $x = -3, y = 5, z = -2$ **D.** $x = -60, y = 60, z = -60$

56. What is the xy-trace of $2x - 4y + z = 8$? **G**

F. $-4y + z = 8$ **G.** $x - 2y = 4$ **H.** $2x + z = 8$ **J.** $z = 8$

Short Response

57. What is the intersection of the xz-traces for the two planes $2x - 3y - 4z = -4$ and $x + 3y + z = 7$? Explain each step of your answer. **See margin p. 151.**

pages 149–150 Exercises

47. The student is actually finding the equation of the y–z–trace. If the student wants the x-intercept, the student should substitute 0 for both y and z in the equation of the plane.

48b. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Mixed Review

GO for Help

Lesson 3-4

58. Maximize the objective function $P = x + 3y$ under the given constraints. At what vertex does this maximum value occur? $\begin{cases} x + y \leq 5 \\ x + 2y \leq 8 \\ x \geq 0, y \geq 0 \end{cases}$
$P = 15$ for $(0, 4)$

Lesson 2-7

Graph each inequality. 59–61. See back of book.

59. $y \leq x - 3$ **60.** $3y - x > -4$ **61.** $2x - y \geq 0$

Lesson 1-6

62. Probability Your town has a drawing for 50 summer jobs. Including you, 150 students apply.
a. What is the probability that you will get one of the jobs? $\frac{1}{3}$
b. You and a friend apply. What is the probability that you both get jobs? about 0.11

Checkpoint Quiz 2 — Lessons 3-4 through 3-5

1. Find the values of x and y that minimize the objective function $C = 2x + 3y$ for the constraints at the right. $\begin{cases} y \geq x \\ x + y \leq 32 \\ x \geq 5, y \geq 3 \end{cases}$
$C = 25$ for $(5, 5)$

2. Agriculture A farmer has at most 400 acres and $45,000 available to grow corn and soybeans. Use the cost and profit information at the right to decide how many acres of each crop will maximize profit.
300 acres corn and 100 acres soybeans

Cost and Profit for Corn and Soybeans

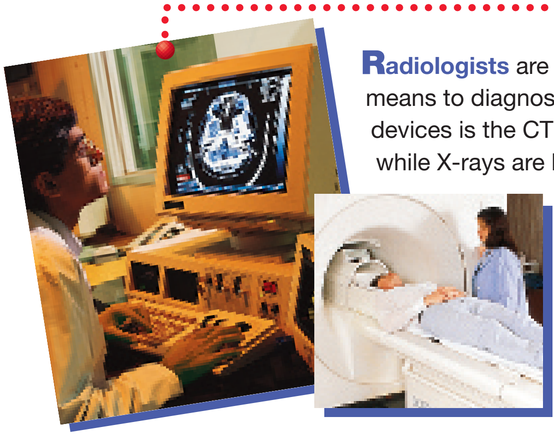
	Corn	Soybeans
Number of Acres	x	y
Cost	$100x$	$150y$
Profit	$60x$	$75y$

Graph each equation. Use traces and intercepts. 3–5. See margin.

3. $3x + 2y + z = 6$ **4.** $x - y + z = 4$ **5.** $4x + y + z = 8$

Algebra at Work

Radiologist



Radiologists are medical doctors who use X-rays, sound waves, and other means to diagnose diseases. Among the radiologist's most powerful diagnostic devices is the CT (computerized tomography) scan. The patient lies on a table while X-rays are beamed through the patient's body from different angles. Images are recorded and fed into a computer. The computer uses a three-dimensional coordinate system to record information and then to produce images of a cross section of the patient's body.

Go Online PHSchool.com **For:** Information about radiology **Web Code:** agb-2031

Checkpoint Quiz

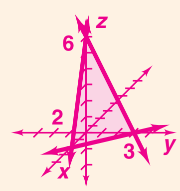
Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 3-4 through 3-5

Resources
Grab & Go
- Checkpoint Quiz 2

57. [2] For the two x–z–traces, let $y = 0$ in each of the equations, which become
$2x - 4z = -4$
$x + z = 7$
One way to solve the system is by elimination:
$2x - 4z = -4$
$4x + 4z = 28$
$6x = 24$
$x = 4$
$z = 3$
Since $y = 0$, the point is (4, 0, 3).
[1] answer only, with no explanation

page 151 Checkpoint Quiz 2

3.



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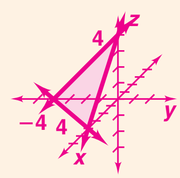

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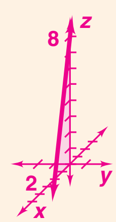


49a. No, a plane that is parallel to two of the axes (and is therefore perpendicular to the third axis) has only two traces, which are perpendicular.

b. No, a plane that intersects two of the axes and is parallel to the third axis has three traces, two of which are parallel.

3-6

1. Plan

Objectives

1 To solve systems in three variables by elimination
2 To solve systems in three variables by substitution

Examples

1 Solving by Elimination
2 Solving an Equivalent System
3 Solving by Substitution
4 Real-World Connection

Professional Development

Math Background

Both the elimination method and the substitution method are used to solve systems of three equations in three variables. There is either no solution, one solution, or infinitely many solutions. Numbering the equations helps to avoid confusion.

More Math Background: p. 116D

Lesson Planning and Resources

See p. 116E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Solving Equations
Lesson 1-3: Example 2
Extra Skills and Word Problems Practice, Ch. 1

Graphing Systems of Equations
Lesson 3-1: Example 1
Extra Skills and Word Problems Practice, Ch. 3

Solving Systems Algebraically
Lesson 3-2: Examples 1, 3–5
Extra Skills and Word Problems Practice, Ch. 3

3-6 Systems With Three Variables

What You'll Learn

- To solve systems in three variables by elimination
- To solve systems in three variables by substitution

. . . And Why

To choose an investment strategy, as in Example 4

Check Skills You'll Need GO for Help Lessons 3-1 and 3-2

Solve each system.

1. $\begin{cases} 2x - y = 11 \\ x + 2y = -7 \end{cases}$ (3, −5) **2.** $\begin{cases} -x + 6y = 8 \\ 2x - 12y = -14 \end{cases}$ no solution **3.** $\begin{cases} 3x + 2y = -5 \\ 4x + 3y = -8 \end{cases}$ (1, −4)

Let $y = 4x - 2$. Solve each equation for x.

4. $3x + y = 5$ 1 **5.** $x - 2y = -3$ 1 **6.** $4x + 3y = 2$ $\frac{1}{2}$

Determine whether the given ordered pair is a solution of each equation in the system.

7. $(1, 3)$ $\begin{cases} 2x + 5y = 17 \\ -4x + 3y = 5 \end{cases}$ yes **8.** $(-4, 2)$ $\begin{cases} x + 2y = 0 \\ 3x - 2y = -16 \end{cases}$ yes

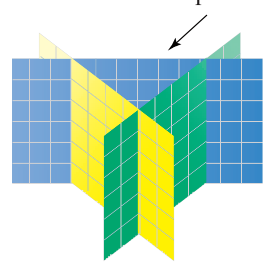
1 Solving Three-Variable Systems by Elimination

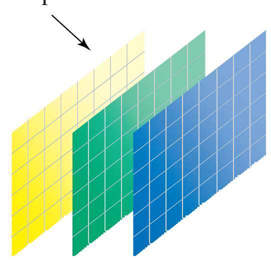
You can represent systems of equations in three variables as graphs in three dimensions. As you learned in Lesson 3-5, the graph of any equation of the form $Ax + By + Cz = D$, where A, B, and C are not all zero, is a plane. The solutions of a three-variable system can be shown graphically as the intersections of planes.

GO for Help

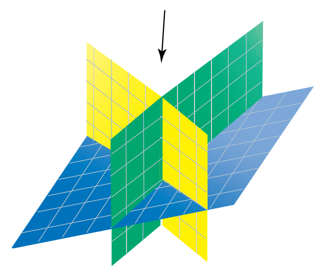
To review solving systems in two variables by graphing, go to Lesson 3-1.

No solution
No point lies in all three planes.

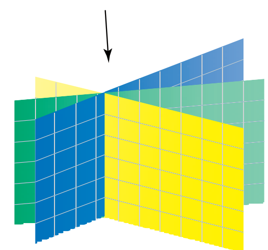




One solution
The planes intersect at one common point.



An infinite number of solutions
The planes intersect at all the points along a common line.



Differentiated Instruction Solutions for All Learners

Special Needs L1
Have students highlight each equation with a different color. Alternatively, students can circle the terms of one variable that are additive inverses, and put a square around the terms of another variable that are additive inverses.
learning style: visual

Below Level L2
When subtracting equations that contain negative coefficients, suggest that students multiply one equation by −1, and then add the equations.
learning style: verbal

When the solution of a system of equations in three variables is represented by one point, you can write it as an ordered triple (x, y, z).

You can solve a system of three equations in three variables by working with the equations in pairs. You will use one of the equations *twice*.

Real-World Connection

You can use a system in three variables to model the amounts of vitamin C, potassium, and beta carotene in this three-fruit salad.

1 EXAMPLE Solving by Elimination

Solve the system by elimination. The equations are numbered to make the procedure easy to follow.

$$\begin{cases} x - 3y + 3z = -4 & ① \\ 2x + 3y - z = 15 & ② \\ 4x - 3y - z = 19 & ③ \end{cases}$$

Step 1 Pair the equations to eliminate y because the y terms are already additive inverses.

$$\begin{array}{l} ①\ x - 3y + 3z = -4 \\ ②\ 2x + 3y - z = 15 \\ \hline ④\ 3x + 2z = 11 \end{array} \quad \textbf{Add.} \quad \begin{array}{l} ②\ 2x + 3y - z = 15 \\ ③\ 4x - 3y - z = 19 \\ \hline ⑤\ 6x - 2z = 34 \end{array}$$

Step 2 Write the two new equations as a system. Solve for x and z.

$$\begin{array}{l} ④\ 3x + 2z = 11 \\ ⑤\ 6x - 2z = 34 \\ \hline 9x = 45 \\ x = 5 \end{array} \qquad \begin{array}{l} ④\ 3x + 2z = 11 \\ 3(5) + 2z = 11 \\ 2z = -4 \\ z = -2 \end{array}$$

Substitute the value of *x*.

Step 3 Substitute the values for x and z into one of the original equations (①, ②, or ③) and solve for y.

$$\begin{aligned} ①\ x - 3y + 3z &= -4 \\ 5 - 3y + 3(-2) &= -4 \\ 5 - 3y - 6 &= -4 \\ -3y &= -3 \\ y &= 1 \end{aligned}$$

The solution of the system is $(5, 1, -2)$.

Check Show that $(5, 1, -2)$ makes each equation true.

$$\begin{array}{ccc} x - 3y + 3z = -4 & 2x + 3y - z = 15 & 4x - 3y - z = 19 \\ 5 - 3(1) + 3(-2) \stackrel{?}{=} -4 & 2(5) + 3(1) - (-2) \stackrel{?}{=} 15 & 4(5) - 3(1) - (-2) \stackrel{?}{=} 19 \\ 5 - 3 - 6 \stackrel{?}{=} -4 & 10 + 3 + 2 \stackrel{?}{=} 15 & 20 - 3 + 2 \stackrel{?}{=} 19 \\ -4 = -4 \checkmark & 15 = 15 \checkmark & 19 = 19 \checkmark \end{array}$$

Quick Check

1 Solve each system by elimination. Check your answers.

a. $\begin{cases} 2x + y - z = 5 \\ 3x - y + 2z = -1 \\ x - y - z = 0 \end{cases}$ **(1, 2, −1)**

b. $\begin{cases} 2x - y + z = 4 \\ x + 3y - z = 11 \\ 4x + y - z = 14 \end{cases}$ **(3, 3, 1)**

c. **Critical Thinking** Suppose, in a three-variable system, the coefficient of one of the variables in one of the equations is zero. When pairing the equations to solve the system, would you use that equation twice? Explain.
No; you wouldn't need to use any of the original equations twice.

You can apply the method in Example 1 to any system of three equations in three variables. You may need to multiply one or more of the equations by a nonzero number to form an equivalent system.

2. Teach

Guided Instruction

1 EXAMPLE Teaching Tip

For Quick Check 1a and 1b, remind students to check their ordered triples by substituting them into *each* equation in the system.

PowerPoint

Additional Examples

1 Solve the system by elimination.

$$\begin{cases} -5x + 3y + 2z = 11 \\ 8x - 5y + 2z = -55 \\ 4x - 7y - 2z = -29 \end{cases}$$

(−2, 5, −7)

Advanced Learners L4

Challenge students to create and solve a system of four equations in four variables. Assure students that they will learn a simpler method in Chapter 4.

learning style: verbal

English Language Learners ELL

Review the geometric terms *point, line,* and *plane* and guide students in recognizing how they are related to the solutions of three-variable systems.

learning style: verbal

2 EXAMPLE Alternative Method

If some students are already familiar with using matrices on graphing calculators to solve equations, they may prefer this method. First students write a matrix that contains the coefficients of the system of equations. To enter the matrix, press **MATRX** > > ENTER 3 ENTER 3 ENTER. Then enter the coefficients. Each row of the matrix represents one equation.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 15 & 6 & -2 \end{bmatrix}$$

Press **QUIT**. Now, write a matrix that contains the constants and enter this as matrix **[B]**. Be sure to press 2 for matrix **[B]** before entering the dimensions.

$$B = \begin{bmatrix} 5 \\ 16 \\ 12 \end{bmatrix}$$

Press **QUIT** again. Tell students that they are going to use the inverse of the coefficient matrix. This is explained in detail in Chapter 4. Now press **MATRX** 1 x^{-1} **MATRX** 2 ENTER. The solution is (−2, 6, −3).

PowerPoint

Additional Examples

2 Solve the system by elimination.

$$\begin{cases} x + 2y + 5z = 1 \\ -3x + 3y + 7z = 4 \\ -8x + 5y + 12z = 11 \end{cases}$$

(−1, −9, 4)

2 EXAMPLE Solving an Equivalent System

Solve the system by elimination.

$$\begin{cases} ① & 2x + y - z = 5 \\ ② & x + 4y + 2z = 16 \\ ③ & 15x + 6y - 2z = 12 \end{cases}$$

Step 1 Pair the equations to eliminate z.

$$\begin{cases} ① & 2x + y - z = 5 \\ ② & x + 4y + 2z = 16 \end{cases} \qquad \begin{aligned} 4x + 2y - 2z &= 10 \quad \textbf{Multiply by 2.} \\ x + 4y + 2z &= 16 \\ ④\ 5x + 6y \quad &= 26 \end{aligned}$$

$$\begin{cases} ② & x + 4y + 2z = 16 \\ ③ & 15x + 6y - 2z = 12 \end{cases}$$
$$⑤ \quad 16x + 10y = 28$$

Step 2 Write the two new equations as a system. Solve for x and y.

$$\begin{cases} ④ & 5x + 6y = 26 \\ ⑤ & 16x + 10y = 28 \end{cases} \qquad \begin{aligned} 25x + 30y &= 130 \quad \textbf{Multiply by 5.} \\ -48x - 30y &= -84 \quad \textbf{Multiply by −3.} \\ -23x \quad &= 46 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} ④ \quad 5x + 6y &= 26 \\ 5(-2) + 6y &= 26 \quad \textbf{Substitute the value of } x. \\ y &= 6 \end{aligned}$$

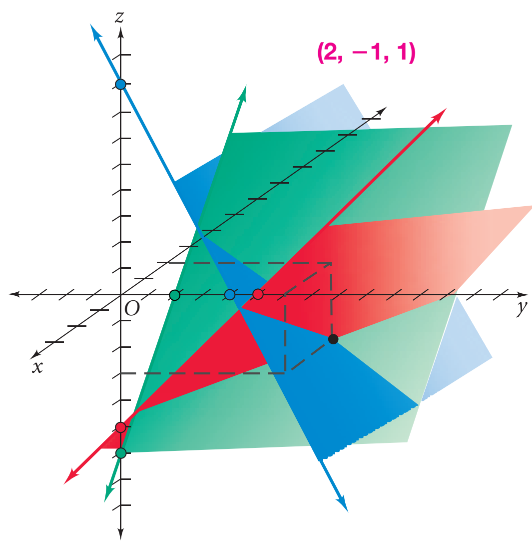
Step 3 Substitute the values for x and y into one of the original equations (①, ②, or ③). Solve for z.

$$\begin{aligned} ① \quad 2x + y - z &= 5 \\ 2(-2) + 6 - z &= 5 \\ z &= -3 \end{aligned}$$

The solution of the system is (−2, 6, −3).

Quick Check 2 Solve the system by elimination. Check your answers.
$$\begin{cases} x + 4y - 5z = -7 \\ 3x + 2y + 3z = 7 \\ 2x + y + 5z = 8 \end{cases}$$

(2, −1, 1)



The graph illustrates the solution to Example 2. Each equation in the system represents a tilted plane. The graph of ① is red, the graph of ② is blue, and the graph of ③ is green. The three planes intersect at (−2, 6, −3).

2 Solving Three-Variable Systems by Substitution

You can also use the substitution method to solve a system of three equations. Substitution is the best method to use when one of the equations can be solved easily for one variable.

3 EXAMPLE Solving by Substitution

GO for Help To review the substitution method, go to Lesson 3-2.

Solve the system by substitution.
$$\begin{cases} x - 2y + z = -4 & ① \\ -4x + y - 2z = 1 & ② \\ 2x + 2y - z = 10 & ③ \end{cases}$$

Step 1 Choose one equation to solve for one of its variables.

① $x - 2y + z = -4$ **Solve the first equation for x.**

$x - 2y = -z - 4$

$x = 2y - z - 4$

Step 2 Substitute the expression for x into each of the other two equations.

② $-4x + y - 2z = 1$

$-4(2y - z - 4) + y - 2z = 1$ **Simplify.**

$-8y + 4z + 16 + y - 2z = 1$

$-7y + 2z + 16 = 1$

④ $-7y + 2z = -15$

③ $2x + 2y - z = 10$

$2(2y - z - 4) + 2y - z = 10$

$4y - 2z - 8 + 2y - z = 10$

$6y - 3z - 8 = 10$

⑤ $6y - 3z = 18$

Step 3 Write the two new equations as a system. Solve for y and z.

$$\begin{cases} -7y + 2z = -15 & ④ \\ 6y - 3z = 18 & ⑤ \end{cases}$$

$-21y + 6z = -45$ **Multiply by 3.**

$12y - 6z = 36$ **Multiply by 2.**

$-9y = -9$

$y = 1$

④ $-7y + 2z = -15$

$-7(1) + 2z = -15$ **Substitute the value of y.**

$-7 + 2z = -15$

$2z = -8$

$z = -4$

Step 4 Substitute the values for y and z into one of the original equations (①, ②, or ③). Solve for x.

① $x - 2y + z = -4$

$x - 2(1) + (-4) = -4$

$x - 2 - 4 = -4$

$x - 6 = -4$

$x = 2$

The solution of the system is (2, 1, −4).

Quick Check 3 Solve each system by substitution. Check your answers.

a. $\begin{cases} x - 3y + z = 6 \\ 2x - 5y - z = -2 \\ -x + y + 2z = 7 \end{cases}$ **(4, 1, 5)**

b. $\begin{cases} 3x + 2y - z = 12 \\ -4x + y - 2z = 4 \\ x - 3y + z = -4 \end{cases}$ **(2, 0, −6)**

Guided Instruction

3 EXAMPLE Teaching Tip

Have students observe that in Step 1, it would also be easy to solve for y (using equation 2) or for z (using equation 3). The amount of work required for solving the system is about the same no matter which variable is used as the basis for the substitution. Use this example to reinforce the idea that there may be a variety of approaches for solving a system. Ask students if they see another approach for this system. If necessary, point out that an efficient approach would be to add equations 1 and 3. Ask students how they would proceed from that point on.

PowerPoint

Additional Examples

3 Solve the system by substitution.
$$\begin{cases} 12x + 7y + 5z = 16 \\ -2x + y - 14z = -9 \\ -3x - 2y + 9z = -12 \end{cases}$$
(7, −9, −1)

4 EXAMPLE Diversity

Some students may come from backgrounds where they have not been exposed to the various types of funds mentioned and some may come from backgrounds where they have. Help all students feel comfortable by explaining what each type of fund is before reading the example.

PowerPoint

Additional Examples

4 You have \$10,000 in a savings account. You want to take most of the money out and invest it in stocks and bonds. You decide to invest nine times as much as you leave in the account. You also decide to invest five times as much in stocks as in bonds. How much will you invest in stocks, how much in bonds, and how much will you leave in savings? **\$7500 in stocks, \$1500 in bonds, \$1000 in savings**

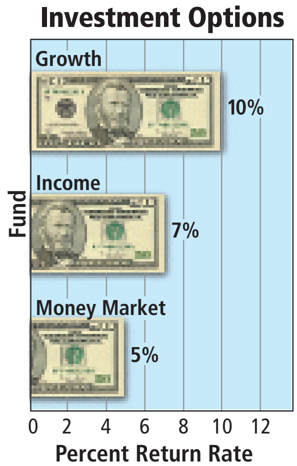
Resources

- Daily Notetaking Guide 3-6 L3
- Daily Notetaking Guide 3-6—Adapted Instruction L1

Closure

Describe what you do to solve a system of three linear equations in three variables by elimination. **Answers may vary. Sample: Multiply one or both of two equations, as needed, to make the coefficients of a selected variable additive inverses. Add to eliminate the variable. Follow the same procedure with a different pair of the original equations to eliminate the same variable. Solve the resulting system of two equations in two variables. Substitute the values you get into one of the original equations to find the value of the third variable.**

4 EXAMPLE Real-World Connection



Money Management Suppose you have saved \$3200 from a part-time job and you want to invest your savings in a growth fund, an income fund, and a money market fund. Refer to the graph. To maximize your return, you decide to put twice as much money in the growth fund as in the money market fund. How should you invest the \$3200 to get a return of \$250 in one year?

Relate growth fund + income fund + money market fund = 3200

growth fund = 2 times money market fund

10% of growth + 7% of income + 5% of money market = 250

Define Let g = amount invested in a growth fund.

Let i = amount invested in an income fund.

Let m = amount invested in a money market fund.

Write
$$\begin{cases} ① & g + i + m = 3200 \\ ② & g = 2m \\ ③ & 0.10g + 0.07i + 0.05m = 250 \end{cases}$$

Step 1 Substitute $2m$ for g in equations ① and ③. Simplify.

$$\begin{aligned} ① \quad g + i + m &= 3200 \\ 2m + i + m &= 3200 \\ ④ \quad 3m + i &= 3200 \end{aligned}$$

$$\begin{aligned} ③ \quad 0.10g + 0.07i + 0.05m &= 250 \\ 0.10(2m) + 0.07i + 0.05m &= 250 \\ ⑤ \quad 0.25m + 0.07i &= 250 \end{aligned}$$

Step 2 Write the two new equations as a system. Solve for m and i.

$$\begin{cases} ④ & 3m + i = 3200 \\ ⑤ & 0.25m + 0.07i = 250 \end{cases}$$

$$\begin{aligned} 3m + i &= 3200 \\ 3m + 0.84i &= 3000 \quad \textbf{Multiply by 12.} \\ 0.16i &= 200 \\ i &= 1250 \end{aligned}$$

$$\begin{aligned} ④ \quad 3m + i &= 3200 \\ 3m + 1250 &= 3200 \quad \textbf{Substitute the value of } i\textbf{.} \\ 3m &= 1950 \\ m &= 650 \end{aligned}$$

Step 3 Substitute the value of m into equation 2 and solve for g.

$$\begin{aligned} ② \quad g &= 2m \\ g &= 2(650) \\ g &= 1300 \end{aligned}$$

You should invest \$1300 in the growth fund, \$1250 in the income fund, and \$650 in the money market fund to get a return of \$250 in one year.

Quick Check

4 Suppose you discover that your growth fund, income fund, and money market fund return rates are better estimated at 12%, 6%, and 3% per year respectively. How should you invest the \$3200 to get a return of \$255 in one year?
\$1400 growth, \$1100 income, and \$700 money market

You have learned how to solve systems of equations using the methods of graphing, elimination, and substitution. In Chapter 4, you will learn how to use matrices to solve systems of equations.

pages 157–159 Exercises

10. (8, −4, 2)

11. $(a, b, c) = (2, 3, -2)$

12. $(r, s, t) = (-2, -1, -3)$

13. (5, 2, 2)

14. (0, 1, 7)

15. (4, 1, 6)

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (pages 153 and 154)

GO for Help

Solve each system by elimination. Check your answers.

1. $\begin{cases} x - y + z = -1 \\ x + y + 3z = -3 \\ 2x - y + 2z = 0 \end{cases}$ (4, 2, −3)

2. $\begin{cases} x - y - 2z = 4 \\ -x + 2y + z = 1 \\ -x + y - 3z = 11 \end{cases}$ (0, 2, −3)

3. $\begin{cases} 2x - y + z = -2 \\ x + 3y - z = 10 \\ x + 2z = -8 \end{cases}$ (2, 1, −5)

4. $\begin{cases} a + b + c = -3 \\ 3b - c = 4 \\ 2a - b - 2c = -5 \end{cases}$ (a, b, c) = (−3, 1, −1)

5. $\begin{cases} 6q - r + 2s = 8 \\ 2q + 3r - s = -9 \\ 4q + 2r + 5s = 1 \end{cases}$ $(q, r, s) = \left(\frac{1}{2}, -3, 1\right)$

6. $\begin{cases} x - y + 2z = -7 \\ y + z = 1 \\ x = 2y + 3z \end{cases}$ (0, 3, −2)

7. $\begin{cases} x + y + 2z = 3 \\ 2x + y + 3z = 7 \\ -x - 2y + z = 10 \end{cases}$ (1, −4, 3)

8. $\begin{cases} 3x - y + z = 3 \\ x + y + 2z = 4 \\ x + 2y + z = 4 \end{cases}$ (1, 1, 1)

9. $\begin{cases} x - 2y + 3z = 12 \\ 2x - y - 2z = 5 \\ 2x + 2y - z = 4 \end{cases}$ (4, −1, 2)

Examples 3 and 4 (pages 155 and 156)

Solve each system by substitution. Check your answers. 10–18. See margin pp. 152–153.

10. $\begin{cases} x + 2y + 3z = 6 \\ y + 2z = 0 \\ z = 2 \end{cases}$

11. $\begin{cases} 3a + b + c = 7 \\ a + 3b - c = 13 \\ b = 2a - 1 \end{cases}$

12. $\begin{cases} 5r - 4s - 3t = 3 \\ t = s + r \\ r = 3s + 1 \end{cases}$

13. $\begin{cases} 13 = 3x - y \\ 4y - 3x + 2z = -3 \\ z = 2x - 4y \end{cases}$

14. $\begin{cases} x + 3y - z = -4 \\ 2x - y + 2z = 13 \\ 3x - 2y - z = -9 \end{cases}$

15. $\begin{cases} x - 4y + z = 6 \\ 2x + 5y - z = 7 \\ 2x - y - z = 1 \end{cases}$

16. $\begin{cases} x - y + 2z = 7 \\ 2x + y + z = 8 \\ x - z = 5 \end{cases}$

17. $\begin{cases} x + y + z = 2 \\ x + 2z = 5 \\ 2x + y - z = -1 \end{cases}$

18. $\begin{cases} 5x - y + z = 4 \\ x + 2y - z = 5 \\ 2x + 3y - 3z = 5 \end{cases}$

Real-World Connection

Careers Stadium designers must take into account the profitability of different types of seating.

19. **Finance** A company placed \$1,000,000 in three different accounts. It placed part in short-term notes paying 4.5% per year, twice as much in government bonds paying 5%, and the rest in utility bonds paying 4%. The income after one year was \$45,500. How much did the company place in each account? See margin.

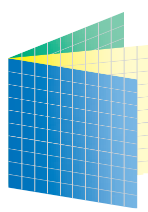
20. **Sports** A stadium has 49,000 seats. Seats sell for \$25 in Section A, \$20 in Section B, and \$15 in Section C. The number of seats in Section A equals the total number of seats in Sections B and C. Suppose the stadium takes in \$1,052,000 from each sold-out event. How many seats does each section hold? See margin.

21. A change machine contains nickels, dimes, and quarters. There are 75 coins in the machine, and the value of the coins is \$7.25. There are 5 times as many nickels as dimes. Find the number of coins of each type in the machine. 50 nickels, 10 dimes, and 15 quarters

B Apply Your Skills

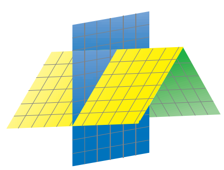
Find the number of solutions of each system.

22.



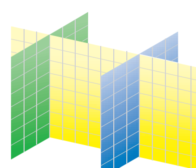
infinitely many solutions

23.



one solution

24.



no solution

3. Practice

Assignment Guide

1 A B 1-9, 22-24, 39-40

2 A B 10-21, 25-38, 41

C Challenge 42-45

Test Prep 46-49

Mixed Review 50-67

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 6, 12, 31, 40, 41.

Error Prevention!

Exercises 5–9 When students need to multiply to eliminate a variable, they may not multiply all terms by the number they choose for the multiplication. Urge students to be careful and thorough in using properties of equality and operations.

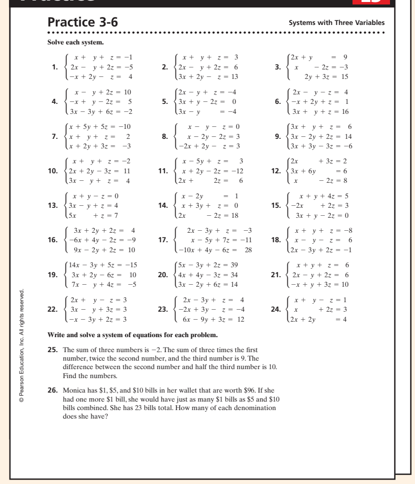
Differentiated Instruction Resources

GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Practice L3



Practice 3-6 Systems with Three Variables

Solve each system.

1. $\begin{cases} x + y + z = -1 \\ 2x - y + 2z = -5 \\ -x + 2y - z = 4 \end{cases}$
2. $\begin{cases} x + y + z = 3 \\ 2x - y + 2z = 6 \\ 3x + 2y - z = 13 \end{cases}$
3. $\begin{cases} 2x + y = 9 \\ x - 2z = -3 \\ 2y + 3z = 15 \end{cases}$
4. $\begin{cases} x - y + 2z = 10 \\ -x + y - 2z = 5 \\ 3x - 3y + 6z = -2 \end{cases}$
5. $\begin{cases} 2x - y + z = -4 \\ 3x + y - 2z = 0 \\ 3x - y = -4 \end{cases}$
6. $\begin{cases} 2x - y - z = 4 \\ -x + 2y + z = 1 \\ 3x + y + z = 16 \end{cases}$
7. $\begin{cases} x + 5y + 5z = -10 \\ x + y + z = 2 \\ x + 2y + 3z = -3 \end{cases}$
8. $\begin{cases} x - y - z = 0 \\ x - 2y - 2z = 3 \\ -2x + 2y - z = 3 \end{cases}$
9. $\begin{cases} 3x + y + z = 6 \\ 3x - 2y + 2z = 14 \\ 3x + 3y - 3z = -6 \end{cases}$
10. $\begin{cases} x + y + z = -2 \\ 2x + 2y - 3z = 11 \\ 3x - y + z = 4 \end{cases}$
11. $\begin{cases} x - 5y + z = 3 \\ x + 2y - 2z = -12 \\ 2x + 2z = 6 \end{cases}$
12. $\begin{cases} 2x + 3z = 2 \\ 3x + 6y = 6 \\ x - 2z = 8 \end{cases}$
13. $\begin{cases} x + y - z = 0 \\ 3x - y + z = 4 \\ 5x + z = 7 \end{cases}$
14. $\begin{cases} x - 2y = 1 \\ x + 3y + z = 0 \\ 2x - 2z = 18 \end{cases}$
15. $\begin{cases} x + y + 4z = 5 \\ -2x + 2z = 3 \\ 3x + y - 2z = 0 \end{cases}$
16. $\begin{cases} 3x + 2y + 2z = 4 \\ -6x + 4y - 2z = -9 \\ 9x - 2y + 2z = 10 \end{cases}$
17. $\begin{cases} 2x - 3y + z = -3 \\ x - 5y + 7z = -11 \\ -10x + 4y - 6z = 28 \end{cases}$
18. $\begin{cases} x + y + z = -8 \\ x - y - z = 6 \\ 2x - 3y + 2z = -1 \end{cases}$
19. $\begin{cases} 14x - 3y + 5z = -15 \\ 3x + 2y - 6z = 10 \\ 7x - y + 4z = -5 \end{cases}$
20. $\begin{cases} 5x - 3y + 2z = 39 \\ 4x + 4y - 3z = 34 \\ 3x - 2y + 6z = 14 \end{cases}$
21. $\begin{cases} x + y + z = 6 \\ 2x - y + 2z = 6 \\ -x + y + 3z = 10 \end{cases}$
22. $\begin{cases} 2x + y - z = 3 \\ 3x - y + 3z = 3 \\ -x - 3y + 2z = 3 \end{cases}$
23. $\begin{cases} 2x - 3y + z = 4 \\ -2x + 3y - z = -4 \\ 6x - 9y + 3z = 12 \end{cases}$
24. $\begin{cases} x + y - z = 1 \\ x + 2z = 3 \\ 2x + 2y = 4 \end{cases}$

Write and solve a system of equations for each problem.

25. The sum of three numbers is −2. The sum of three times the first number, twice the second number, and the third number is 9. The difference between the second number and half the third number is 10. Find the numbers.

26. Monica has \$1, \$5, and \$10 bills in her wallet that are worth \$96. If she had one more \$1 bill, she would have just as many \$1 bills as \$5 and \$10 bills combined. She has 23 bills total. How many of each denomination does she have?

16. (5, −2, 0)

17. (1, −1, 2)

18. (1, 3, 2)

19. \$220,000 was placed in short-term notes. \$440,000 was placed in government bonds. \$340,000 was placed in utility bonds.

20. Section A has 24,500 seats. Section B has 14,400 seats. Section C has 10,100 seats.

4. Assess & Reteach

PowerPoint

Lesson Quiz

Solve each system by elimination.

1. $\begin{cases} -3x + 2y - 5z = -3 \\ 3x - y + 3z = 4 \\ 3x + 5y - 8z = 6 \end{cases}$

   (3, −7, −4)

2. $\begin{cases} 7x - y - z = 2 \\ 13x - 4y + 5z = -7 \\ -4x + 3y - 4z = -5 \end{cases}$

   (−2, −11, −5)

3. Solve by substitution.

   $\begin{cases} 2x - 3y + 6z = -21 \\ -5x + 4y + z = 3 \\ 7x - 7y - 4z = -6 \end{cases}$

   (3, 5, −2)

Alternative Assessment

Ask a student to state a three-variable equation. Ask another student to state a three variable equation that contains an additive inverse of one of the terms in the first equation. Ask another student to state an equation that has a multiple of the additive inverse. Write all of the equations where they can be seen as the students give them to you. Instruct all members of the class to solve the system. Students may want to work in pairs.

Solve each system.

25. $\begin{cases} x - 3y + 2z = 11 \\ -x + 4y + 3z = 5 \\ 2x - 2y - 4z = 2 \end{cases}$ (8, 1, 3)

26. $\begin{cases} x + 2y + z = 4 \\ 2x - y + 4z = -8 \\ -3x + y - 2z = -1 \end{cases}$ (3, 2, −3)

27. $\begin{cases} 4x - y + 2z = -6 \\ -2x + 3y - z = 8 \\ 2y + 3z = -5 \end{cases}$ $\left(\frac{1}{2}, 2, -3\right)$

28. $\begin{cases} 4A + 2U + I = 2 \\ 5A - 3U + 2I = 17 \\ A - 5U = 3 \end{cases}$ (−2, −1, 12)

29. $\begin{cases} 4x - 2y + 5z = 6 \\ 3x + 3y + 8z = 4 \\ x - 5y - 3z = 5 \end{cases}$ no solution

30. $\begin{cases} 2\ell + 2w + h = 72 \\ \ell = 3w \\ h = 2w \end{cases}$ (21.6, 7.2, 14.4)

31. $\begin{cases} 3x + 2y - z = 17.8 \\ x - 3y + 2z = 7.9 \\ 2x + y - 3z = 3.9 \end{cases}$ (6, 1.5, 3.2)

32. $\begin{cases} x + 2y = 2 \\ 2x + 3y - z = -9 \\ 4x + 2y + 5z = 1 \end{cases}$ $\left(-\frac{122}{11}, \frac{72}{11}, \frac{71}{11}\right)$

33. $\begin{cases} 3x + 2y + 2z = -2 \\ 2x + y - z = -2 \\ x - 3y + z = 0 \end{cases}$ $\left(-\frac{10}{13}, -\frac{2}{13}, \frac{4}{13}\right)$

34. $\begin{cases} 6x + y - 4z = -8 \\ \frac{y}{4} - \frac{z}{6} = 0 \\ 2x - z = -2 \end{cases}$ (2, 4, 6)

35. $\begin{cases} 4y + 2x = 6 - 3z \\ x + z - 2y = -5 \\ x - 2z = 3y - 7 \end{cases}$ (−1, 2, 0)

36. $\begin{cases} 5z + 4y = 4 \\ 3x - 2y = 0 \\ x + 3z = -8 \end{cases}$ (4, 6, −4)

37. $\begin{cases} x + 6z = 12 \\ -2x + 3y = 6 \\ y - \frac{z}{2} = \frac{5}{2} \end{cases}$ $\left(2, \frac{10}{3}, \frac{5}{3}\right)$

38. $\begin{cases} 4x - y + z = -5 \\ -x + y - z = 5 \\ 2x - z - 1 = y \end{cases}$ (0, 2, −3)

History **Exercises 39 and 40 appeared in the book *Algebraical Problems*, published in 1824. Write and solve a system for each problem.**

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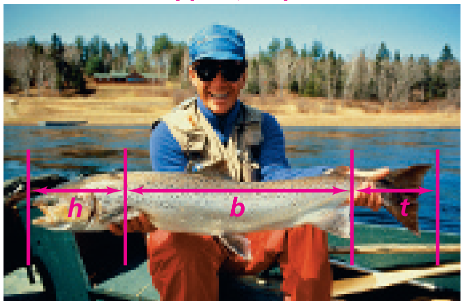
39. Ten apples cost a penny, and 25 pears cost two pennies. Suppose I buy 100 apples and pears for $9\frac{1}{2}$ pennies. How many of each shall I have? 75 apples; 25 pears

40. A fish was caught whose tail weighed 9 lb. Its head weighed as much as its tail plus half its body. Its body weighed as much as its head and tail. What did the fish weigh? 72 pounds

GPS

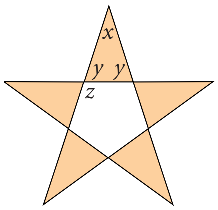


41. **Writing** How do you decide whether substitution is the best method to solve a system in three variables? See margin.

Challenge

42. **Error Analysis** A student says that the system consisting of $x = 0$, $y = 0$, and $z = 0$ has no solutions. Explain the student's error. See margin.

43. **Geometry** Refer to the regular five-pointed star at the right. Write and solve a system of three equations to find the measure of each labeled angle. See above left.



Problem Solving Hint

Sum of angles in a pentagon: 540°

44. **Open-Ended** Write your own system having three variables. Begin by choosing the solution. Then write three equations that are true for your solution. Use elimination to solve the system. See margin.

43. $x + 2y = 180$
$y + z = 180$
$5z = 540$
$x = 36, y = 72, z = 108$

pages 157–158 Exercises

41. Answers may vary. Sample: When one of the equations can easily be solved for one variable, it is easier to use substitution.

42. The student is thinking that 0 means that there is no solution. The point (0, 0, 0) is the solution.

44. Answers may vary. Sample: Solution is (1, 2, 3)
$x + y + 2 = 6$
$2x - y + 2z = 6$
$3x + 3y + z = 12$

45. Let E, F, and V represent the numbers of edges, faces, and vertices, respectively. From the statement Every face has five edges, and the number of edges is 5 times the number of faces: $E = 5F$. But since each edge is part of two faces, this counts each edge twice. So $E = \frac{5}{2}F$. Since every face has five vertices and every vertex is shared by three faces, $3V = 5F$ or $V = \frac{5}{3}F$. Euler's formula: $V + F = E + 2$. Solving this system of three equations yields $E = 30$, $F = 12$, and $V = 20$

45. **Geometry** In the regular polyhedron described below, all faces are congruent polygons. Use a system of three linear equations to find the numbers of vertices, edges, and faces. See margin p. 158.

Every face has five edges and every edge is shared by two faces. Every face has five vertices and every vertex is shared by three faces. The sum of the number of vertices and faces is two more than the number of edges.

Test Prep

Multiple Choice

46. What is the solution of the system? $\begin{cases} -3x + 2y - z = 6 \\ 3x + y + 2z = 5 \\ 2x - 2y - z = -5 \end{cases}$ **B**

A. $(6, 5, -3)$ B. $(1, 4, -1)$
C. $\left(0, \frac{17}{5}, \frac{4}{5}\right)$ D. no solution

47. What is the solution of the system? $\begin{cases} x + 3y - 2z = -8 \\ 3x - y + z = 11 \\ 2x + 4y + 2z = 14 \end{cases}$ **F**

F. $(2, 0, 5)$ G. $(-8, 11, 14)$
H. $\left(-2, \frac{4}{3}, 5\right)$ J. no solution

48. What is the solution of the system? $\begin{cases} y = -2x + 10 \\ -x + y - 2z = -2 \\ 3x - 2y + 4z = 7 \end{cases}$ **D**

A. $\left(3, -4, \frac{3}{2}\right)$ B. $\left(3, 16, \frac{15}{2}\right)$
C. $\left(-3, 16, \frac{15}{2}\right)$ D. $\left(3, 4, \frac{3}{2}\right)$

Short Response

49. Why is there no solution to the system? Explain your answer in terms of intersecting planes. See back of book.

$\begin{cases} ① \; 2x - 3y + z = 5 \\ ② \; 2x - 3y + z = -2 \\ ③ \; -4x + 6y - 2z = 10 \end{cases}$

Mixed Review

GO for Help

Lesson 3-5 Graph each equation. 50–52. See margin. 53–55. See back of book.

50. $x + y + 4z = 8$ 51. $2x + 3y - z = 12$ 52. $-3x + y + 5z = 15$
53. $-2x + 3y - z = 6$ 54. $6x + 4y - 3z = -12$ 55. $3x - 6y - 2z = 18$

Lesson 2-5 Graph each equation. 56–61. See margin.

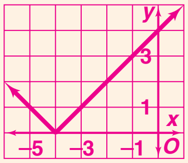
56. $y = |x + 4|$ 57. $y = |3x - 2|$ 58. $y = \left|\frac{1}{2}x + 3\right| - 2$
59. $y = |x - 2| + 1$ 60. $y = |2x + 1|$ 61. $y = |x + 3| - 2$

Lesson 1-4 Solve each inequality. Graph the solution on a number line. 62–67. See back of book.

62. $-4x + 3 \le 9$ 63. $-(x + 4) - 3 \ge 11$ 64. $2(3x - 1) < x - 7$
65. $6 - 2x > 2$ 66. $3x + 2 < -x + 10$ 67. $-2(x + 3) \ge x$

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies

56.


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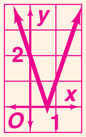

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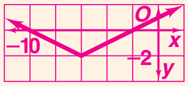

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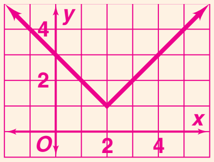

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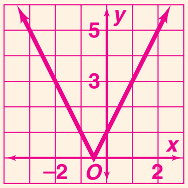

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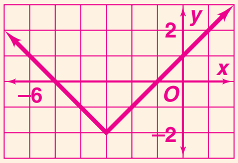

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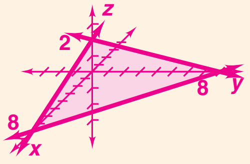

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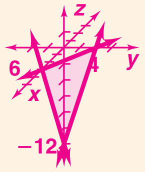

52.