3-1

1. Plan

Objectives
1 To solve a system by graphing

Examples
1 Solving by Graphing
2 Real-World Connection
3 Classifying Systems Without Graphing

Professional Development

Math Background

When you solve a system of equations by graphing, the *x*- and *y*-coordinates give an estimated point of intersection which can help verify solutions found using the methods in the following lessons.

More Math Background: p. 116C

Lesson Planning and Resources

See p. 116E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Linear Equations
Lesson 2-2: Example 1
Extra Skills and Word Problems Practice, Ch. 2

Graphing Systems of Equations

What You'll Learn
- To solve a system by graphing

. . . And Why
To make predictions from data, as in Example 2

Check Skills You'll Need GO for Help Lesson 2-2

Graph each equation. 1–6. See back of book.

1. $y = 3x - 2$
2. $y = -x$
3. $y = -\frac{1}{2}x + 4$

Graph each equation. Use one coordinate plane for all three graphs.

4. $2x - y = 1$
5. $2x - y = -1$
6. $x + 2y = 2$

New Vocabulary
- system of equations
- linear system
- independent system
- dependent system
- inconsistent system

1 Systems of Linear Equations

Technology Activity: Analyzing Graphs

1. Use a graphing calculator to graph each pair of equations. 1a–c. See back of book.
   - **a.** $y = x + 5$, $y = -2x + 5$
   - **b.** $y = 3x + 2$, $y = 3x - 1$
   - **c.** $y = -4x - 2$, $y = \frac{8x + 4}{-2}$
2. For each pair, answer the following questions. a–b. See back of book.
   - **a.** Do the graphs have any points in common; if so, how many?
   - **b.** Compare the slopes of the graphs. What is the relationship between the slopes and the number of points in common?
3. Copy and complete the table for the graphs of two linear equations.

Description of Lines	How Many Points of Intersection?	Equal Slopes? (yes/no)	Same *y*-intercepts? (yes/no)
intersecting	1	no	either
parallel	0	yes	no
coinciding	infinite	yes	yes

For: Linear Systems Activity
Use: Interactive Textbook, 3-1

A **system of equations** is a set of two or more equations that use the same variables. If the graph of each equation in a system of two variables is a line, then the system is a **linear system.**

A brace is used to keep the equations of a system together.

$$\begin{cases} y = x + 3 \\ y = -2x + 3 \end{cases}$$

Differentiated Instruction Solutions for All Learners

Special Needs L1
Have students identify and touch three different pairs of lines suggested by objects in the classroom: intersecting lines, parallel lines, and lines that coincide.
learning style: tactile

Below Level L2
Review the concept of slope. Have students hold their pencils in the air and slowly rotate them to visualize slope as a measure of steepness. Review positive and negative slope and how to graph a line in slope-intercept form, such as $y = \frac{1}{2}x + 1$ and $y = 3x - 2$.
learning style: visual

A solution of a system of equations is a set of values for the variables that makes all the equations true. You can solve some linear systems by graphing the equations. The points where both (or all) the graphs intersect represent solutions.

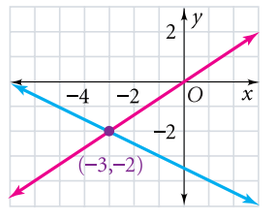
1 EXAMPLE Solving by Graphing

Multiple Choice Which ordered pair of numbers is the solution of the system?

$$\begin{cases} x + 2y = -7 \\ 2x - 3y = 0 \end{cases}$$

Ⓐ (1, −4) Ⓑ (−1, −3) Ⓒ (−6, −4) Ⓓ (−3, −2)

Graph the equations and find the intersection. The solution appears to be (−3, −2).



Check Show that (−3, −2) makes both equations true.

$x + 2y = -7$ | $2x - 3y = 0$

$-3 + 2(-2) \stackrel{?}{=} -7$ | $2(-3) - 3(-2) \stackrel{?}{=} 0$

$-3 - 4 \stackrel{?}{=} -7$ | $-6 + 6 \stackrel{?}{=} 0$

$-7 = -7$ ✔ | $0 = 0$ ✔

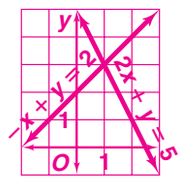
The answer is D.

Test-Taking Tip

You can also substitute values from each choice into the equations. But be careful. Some choices will make one equation true but not the other.

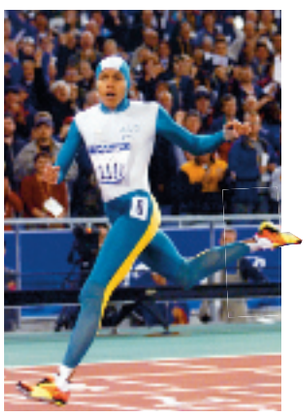
Quick Check

1 Solve $\begin{cases} 2x + y = 5 \\ -x + y = 2 \end{cases}$ by graphing. Check your solution. **(1, 3)**



2 EXAMPLE Real-World Connection

Sports Winning times for the Olympic 400-m run have been decreasing more rapidly for women than for men. Use the data in the table to find linear models for women's and men's times. Assuming that these trends continue, predict the year in which the women's winning time could equal that of the men.



Real-World Connection

Cathy Freeman won the 400-m run in the 2000 Olympics.

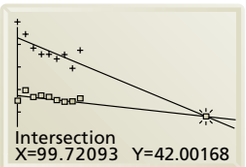
Winning Times for the Olympic 400-Meter Dash (seconds)

Year	1968	1972	1976	1980	1984	1988	1992	1996	2000
Men's Time	43.86	44.66	44.26	44.60	44.27	43.87	43.50	43.49	43.84
Women's Time	52.03	51.08	49.29	48.88	48.83	48.65	48.83	48.25	49.11

SOURCE: *The World Almanac*

Step 1 Let x = number of years since 1968.
Let y = winning times in seconds.
Use the **LinReg** feature of a graphing calculator to find linear models.
Men's time: $y \approx -0.02433x + 44.43$
Women's time: $y \approx -0.08883x + 50.86$

Step 2 Graph each model. Use the Intersect feature on the graphing calculator. The two lines meet at about (99.7, 42.0).



If the trends continue, the times for men and women will be equal about 100 years from 1968, in 2068.

Quick Check

2a. 2008 men: 43.46 s, women: 47.31 s
2024 men: 43.07 s, women: 45.89 s

2 **a.** Use the models in Example 2 to predict the winning times for the 400-m run at the Olympics in 2008 and in 2024.

b. Critical Thinking Example 2 assumes that current trends will continue. Explain why that assumption may not be valid. **It would mean that eventually the 400-m run would be run in zero seconds, and this could not happen.**

Advanced Learners L4
Have students explain why linear systems must be either independent, dependent, or inconsistent.
The graphs intersect, coincide, or are parallel.
learning style: verbal

English Language Learners ELL
Some students may have difficulty distinguishing between the terms *independent, dependent,* and *inconsistent.* Have students sketch and label graphs for each corresponding system and its number of solutions. Have students use their diagrams as a reference.
learning style: visual

2. Teach

Guided Instruction

Technology Activity
Some students may think that all lines with equal slopes are parallel. Remind them that the lines might coincide.

1 EXAMPLE Teaching Tip

Point out that when the point of intersection does not have integer coordinates, a graphing calculator may be helpful.

2 EXAMPLE Connection to Physical Education

Every year, more and more girls are joining sports teams. Many are breaking into sports such as football that have generally been thought of as "for boys only."

PowerPoint Additional Examples

1 Solve the system by graphing.

$$\begin{cases} x + 3y = 2 \\ 3x + 3y = -6 \end{cases}$$ **(−4, 2)**

2 The table shows the number of pairs of shoes sold by two new employees at a shoe store. Find linear models for each employee's sales. Use the graph of the models to predict the week in which they could sell the same number of pairs of shoes.

Week	1	2	3	4
Ed	50	55	63	67
Jo	40	47	56	62

Ed: $y = 5.9x + 44$;
Jo: $y = 7.5x + 32.5$;
during week 8

Additional Examples

3 Classify the system without graphing.

$\begin{cases} y = 3x + 2 \\ -6x + 2y = 4 \end{cases}$

dependent system

Resources

- Daily Notetaking Guide 3-1 L3
- Daily Notetaking Guide 3-1—Adapted Instruction L1

Closure

Ask students to describe the graph of an independent system of linear equations in two variables. **The graph consists of two lines that intersect in one point.**

Vocabulary Tip

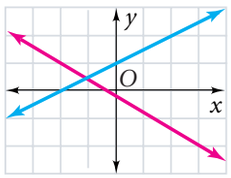
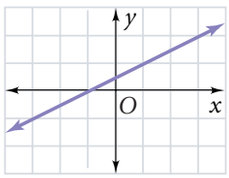
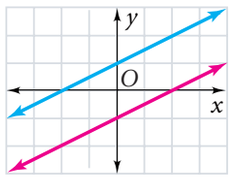
Unique means "exactly one."

You can classify a system of two linear equations by the number of solutions. A system that has a unique solution, as in Examples 1 and 2, is an **independent system.** However, not every system has a unique solution.

A **dependent system** does not have a unique solution. An **inconsistent system** is a system that has no solution.

Key Concepts

Summary **Graphical Solutions of Linear Systems in Two Variables**

Intersecting Lines	Coinciding Lines	Parallel Lines
		
one solution	no unique solution	no solution
Independent	**Dependent**	**Inconsistent**

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You can also classify a system of equations without graphing. By comparing the slopes and y-intercepts of the equations, you can find the number of solutions.

3 EXAMPLE Classifying Systems Without Graphing

Classify the system without graphing. $\begin{cases} y = 2x + 3 \\ -2x + y = 1 \end{cases}$

$y = 2x + 3$ **Rewrite in slope-intercept form.** → $y = 2x + 1$

$m = 2, b = 3$ ← **Find the slope and y-intercept.** → $m = 2, b = 1$

Since the slopes are the same, the lines could coincide. Compare the y-intercepts. Since the y-intercepts are different, the lines are parallel. There is no solution. The system is an inconsistent system.

Quick Check 3 Without graphing, classify each system as *independent*, *dependent*, or *inconsistent*.

a. $\begin{cases} 3x + y = 5 \\ 15x + 5y = 2 \end{cases}$ inconsistent

b. $\begin{cases} y = 2x + 3 \\ -4x + 2y = 6 \end{cases}$ dependent

c. $\begin{cases} x - y = 5 \\ y + 3 = 2x \end{cases}$ independent

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 119)

GO for Help

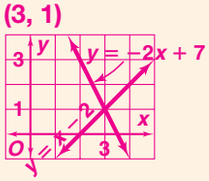
Solve each system by graphing. Check your answers. 1–5. See margin.

1. $\begin{cases} y = x - 2 \\ y = -2x + 7 \end{cases}$
2. $\begin{cases} y = -x + 3 \\ y = \frac{3}{2}x - 2 \end{cases}$
3. $\begin{cases} 2x + 4y = 12 \\ x + y = 2 \end{cases}$
4. $\begin{cases} x = -3 \\ y = 5 \end{cases}$
5. $\begin{cases} 2x - 2y = 4 \\ y - x = 6 \end{cases}$
6. $\begin{cases} 3x + y = 5 \\ x - y = 7 \end{cases}$
7. $\begin{cases} -5x + y = -9 \\ x + 3y = 21 \end{cases}$
8. $\begin{cases} y = x \\ y - 5x = 0 \end{cases}$
9. $\begin{cases} x = 10 \\ x = y - 10 \end{cases}$

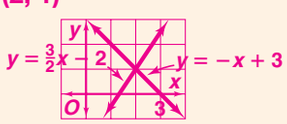
6–9. See back of book.

pages 120–123 Exercises

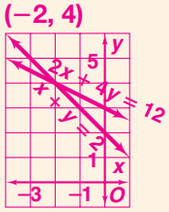
1. (3, 1)



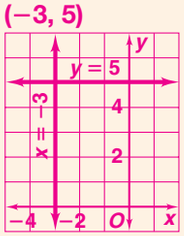
2. (2, 1)



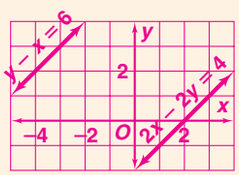
3. (−2, 4)



4. (−3, 5)



5. no solution



12a. Anwers may vary.

$\begin{cases} y = 3000x + 5200 \\ y = -900x + 35{,}700 \end{cases}$

b. If Feb = 1, the revenue will equal expenses in the 7.82 month or August.

Example 2 (page 119)

For Exercises 10–11, use your graphing calculator. Find linear models for each set of data. Use each model to predict the year in which the quantities will be equal.

10. Annual U.S. Consumption of Vegetables

Year	Broccoli (lb/person)	Cucumbers (lb/person)
1980	1.5	3.9
1985	2.6	4.4
1990	3.4	4.7
1995	4.3	5.6
1998	5.1	6.5
1999	6.5	6.8
2000	6.1	6.4

SOURCE: *Statistical Abstract of the United States.* Go to **www.PHSchool.com** for a data update. Web Code: agg-9041

11. U.S. Life Expectancy at Birth

Year	Men (years)	Women (years)
1970	67.1	74.7
1975	68.8	76.6
1980	70.0	77.4
1985	71.1	78.2
1990	71.8	78.8
1995	72.5	78.9
2000	74.3	79.7

SOURCE: U.S. Census Bureau. Go to **www.PHSchool.com** for a data update. Web Code: agg-9041

10–11. Models may vary. Samples are given

10. Use 0 for 1980.

$\begin{cases} y = 0.232x + 1.328 \\ y = 0.145x + 3.673 \end{cases}$

about 2007

11. Use 0 for 1970.

$\begin{cases} y = 0.22x + 67.5 \\ y = 0.15x + 75.507 \end{cases}$

about 2185

12. a. Business The spreadsheet shows the monthly revenue and monthly expenses for a new business. Find a linear model for monthly revenue and a linear model for monthly expenses. **a–b. See margin p. 120.**

b. Use the models to predict the month in which revenue will equal expenses.

	A	B	C
1	Month	Revenue	Expenses
2	Feb	8000	35000
3	Mar	12000	33000
4	Apr	13000	34000
5	May	18000	32000
6	Jun	20000	31000

Example 3 (page 120)

Without graphing, classify each system as *independent*, *dependent*, or *inconsistent*.

13. $\begin{cases} 7x - y = 6 \\ -7x + y = -6 \end{cases}$ dependent

14. $\begin{cases} -3x + y = 4 \\ x - \frac{1}{3}y = 1 \end{cases}$ inconsistent

15. $\begin{cases} 4x + 8y = 12 \\ x + 2y = -3 \end{cases}$ inconsistent

16. $\begin{cases} y = 2x - 1 \\ y = -2x + 5 \end{cases}$ independent

17. $\begin{cases} x = 6 \\ x = -2 \end{cases}$ inconsistent

18. $\begin{cases} 2y = 5x + 6 \\ -10x + 4y = 8 \end{cases}$ inconsistent

19. $\begin{cases} x - 3y = 2 \\ 4x - 12y = 8 \end{cases}$ dependent

20. $\begin{cases} x + 4y = 12 \\ 2x - 8y = 4 \end{cases}$ independent

21. $\begin{cases} 4x + 8y = -6 \\ 6x + 12y = -9 \end{cases}$ dependent

22. $\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$ inconsistent

23. $\begin{cases} y - x = 0 \\ y = -x \end{cases}$ independent

24. $\begin{cases} 2y - x = 4 \\ \frac{1}{2}x - y = 2 \end{cases}$ inconsistent

B Apply Your Skills

25–36. See back of book.

Graph and solve each system. Where necessary, estimate the solution.

25. $\begin{cases} 3 = 4y + x \\ 4y = -x + 3 \end{cases}$

26. $\begin{cases} x - 2y + 1 = 0 \\ x + 4y - 6 = 0 \end{cases}$

27. $\begin{cases} 3x + 6y - 12 = 0 \\ x + 2y = 8 \end{cases}$

28. $\begin{cases} -x + 3y = 6 \\ 2x - y = 8 \end{cases}$

29. $\begin{cases} 3x + y = 3 \\ 2x - y = 7 \end{cases}$

30. $\begin{cases} 2x + 3y = 6 \\ 4x = 6y + 3 \end{cases}$

31. $\begin{cases} 10 - 3x = -3y \\ 2 = 2x + y \end{cases}$

32. $\begin{cases} 3x = -5y + 4 \\ 250 + 150x = 300 \end{cases}$

33. $\begin{cases} x + 3y = 6 \\ 6y + 2x = 12 \end{cases}$

34. $\begin{cases} 2y + x = 8 \\ y - 2x = -6 \end{cases}$

35. $\begin{cases} y = -2x + 6 \\ x - 3y = -6 \end{cases}$

36. $\begin{cases} -x - 2 = -2y \\ 2x - 4y - 4 = 0 \end{cases}$

3. Practice

Assignment Guide

A B	1-50
C Challenge	51-55
Test Prep	56-61
Mixed Review	62-74

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 6, 12, 39, 45, 50.

Exercise 5 If students draw the graphs correctly, the lines should look parallel. Be sure students understand how slope can be used to confirm that the lines really are parallel.

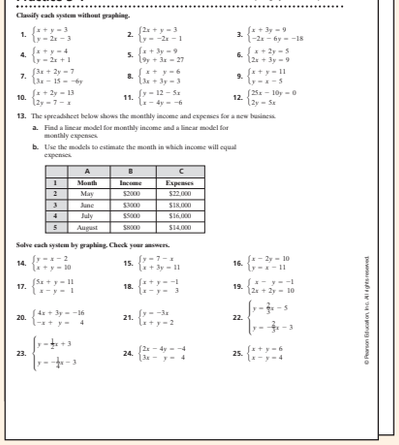
Differentiated Instruction Resources

GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Practice L3



Practice 3-1 Graphing Systems of Equations

Classify each system without graphing.

1. $\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$
2. $\begin{cases} 2x + y = 3 \\ y = -2x - 1 \end{cases}$
3. $\begin{cases} x + 3y = 9 \\ -2x + 6y = -18 \end{cases}$
4. $\begin{cases} x + y = 4 \\ y = 2x + 1 \end{cases}$
5. $\begin{cases} x + 3y = 9 \\ 9y + 3x = 27 \end{cases}$
6. $\begin{cases} x + 2y = 5 \\ 2x + 3y = 9 \end{cases}$
7. $\begin{cases} 3x + 2y = 7 \\ 3x - 15 = -6y \end{cases}$
8. $\begin{cases} x + y = 6 \\ 3x + 3y = 3 \end{cases}$
9. $\begin{cases} x + y = 11 \\ y = x - 5 \end{cases}$
10. $\begin{cases} x + 2y = 13 \\ 2y = 7 - x \end{cases}$
11. $\begin{cases} y = 12 - 5x \\ x - 4y = -6 \end{cases}$
12. $\begin{cases} 25x - 10y = 0 \\ 2y = 5x \end{cases}$

13. The spreadsheet below shows the monthly income and expenses for a new business.
   a. Find a linear model for monthly income and a linear model for monthly expenses.
   b. Use the models to estimate the month in which income will equal expenses.

	A	B	C
1	Month	Income	Expenses
2	May	$2000	$22,000
3	June	$3000	$18,000
4	July	$5000	$16,000
5	August	$8000	$14,000

Solve each system by graphing. Check your answers.

14. $\begin{cases} y = x - 2 \\ x + y = 10 \end{cases}$
15. $\begin{cases} y = 7 - x \\ x + 3y = 11 \end{cases}$
16. $\begin{cases} x - 2y = 10 \\ y = x - 11 \end{cases}$
17. $\begin{cases} 5x + y = 11 \\ x - y = 1 \end{cases}$
18. $\begin{cases} x + y = -1 \\ x - y = 3 \end{cases}$
19. $\begin{cases} x - y = -1 \\ 2x + 2y = 10 \end{cases}$
20. $\begin{cases} 4x + 3y = -16 \\ -x + y = 4 \end{cases}$
21. $\begin{cases} y = -3x \\ x + y = 2 \end{cases}$
22. $\begin{cases} y = \frac{2}{3}x - 5 \\ y = -\frac{2}{3}x - 3 \end{cases}$
23. $\begin{cases} y = \frac{1}{2}x + 3 \\ y = -\frac{1}{2}x - 3 \end{cases}$
24. $\begin{cases} 2x - 4y = -4 \\ 3x - y = 4 \end{cases}$
25. $\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$

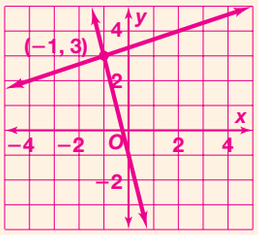
4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Graph and solve the system.

$\begin{cases} 4x + y = -1 \\ -x + 3y = 10 \end{cases}$ **(−1, 3)**



Classify each system without graphing. Tell how many solutions there are.

2. $\begin{cases} 5x + 3y = 10 \\ -x - 0.6y = -2 \end{cases}$
dependent; infinitely many

3. $\begin{cases} 12x - 18y = 9 \\ -6x + 9y = 13 \end{cases}$
inconsistent; no solutions

4. $\begin{cases} 4x + 5y = -10 \\ 3x - 8y = 15 \end{cases}$
independent; one solution

Alternative Assessment

Have students work in groups of three. Students 1 and 2 each write an equation of the form $Ax + By = C$, where A, B, and C are non-zero integers from -9 to 9. Student 3 classifies and graphs the system. If there is a unique solution with integer x- and y-values, student 3 identifies the solution and checks that it is correct. Otherwise, student 3 estimates the solution. Students 1 and 2 then confirm the correctness of the work. Students change roles until each student has graphed two systems.

Exercise 53 Check that students understand what the solution set notation means geometrically.

Real-World Connection

Groomers must be able to handle dogs of every breed and temperament.

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Challenge

37. Banking To pay your monthly bills, you can either open a checking account or use an online banking service. A local bank charges \$3 per month and \$.40 per check, while an online services charges a flat fee of \$9 per month.
a. Write and graph a system of linear equations to model the cost c of each service for b bills that you need to pay monthly. **See back of book.**
b. Find the point of intersection of the two linear models. What does this answer represent? **b–c. See margin.**
c. If you pay about 12 bills per month, which service should you choose? Explain.

Classify each system without graphing.

38. $\begin{cases} 3x - 2y = 8 \\ 4y = 6x - 5 \end{cases}$ **inconsistent**

39. $\begin{cases} 2x + 8y = 6 \\ x = -4y + 3 \end{cases}$ **dependent**

40. $\begin{cases} 3a + 6b = 14 \\ -a + 2b = 3 \end{cases}$ **independent**

41. $\begin{cases} 3m = -5n + 4 \\ n - \frac{6}{5} = -\frac{3}{5}m \end{cases}$ **inconsistent**

42. $\begin{cases} -12x + 4y = 8 \\ y - 4 = 3x \end{cases}$ **inconsistent**

43. $\begin{cases} -6y + 18 = 12x \\ 3y + 6x = 9 \end{cases}$ **dependent**

44. Fees Suppose you are going on vacation and leaving your dog in a kennel. The Bowowery charges \$25 per day, which includes a one-time grooming treatment. The Poochpad charges \$20 per day and a one-time fee of \$30 for grooming.
a. Write a system of equations to represent the cost c for d days that your dog will stay at a kennel. **a, c. See margin p. 123.**
b. Using a graphing calculator, find the number of days for which the costs are the same. **See back of book.**
c. If your vacation is a week long, which kennel should you choose? Explain.

45. Advertising GPS You and your business partner are mailing advertising flyers to your customers. You address 6 flyers each minute and have already done 80. Your partner addresses 4 flyers each minute and has already done 100. Graph and solve a system of equations to find when the two of you will have addressed equal numbers of flyers. **See back of book.**

Open-Ended Write a second equation for each system so that the system will have the indicated number of solutions. **46–48. Answers may vary. Samples:**

46. one
$\begin{cases} y = -3x + 2 \\ ? \end{cases}$
$y = x + 3$

47. none
$\begin{cases} y = -4x - 6 \\ ? \end{cases}$
$y = -4x + 8$

48. an infinite number
$\begin{cases} 3y = 6x + 7 \\ ? \end{cases}$
$y = 2x + \frac{7}{3}$

49. Reasoning Is it possible for an inconsistent linear system to consist of two lines with the same y-intercept? Explain. **No; they would be the same line, and the system would be dependent and consistent.**

50. Writing Summarize the possible relationships for the y-intercepts, slopes, and number of solutions in a system of two linear equations of two variables. **See margin p. 123.**

Open-Ended Write a second equation for each system so that the system will have the indicated number of solutions. **51–52. Answers may vary. Samples:**

51. infinite number of solutions
$\begin{cases} \frac{x}{4} + \frac{y}{3} = 1 \\ ? \end{cases}$
$3x + 4y = 12$

52. no solutions
$\begin{cases} 5x + 2y = 10 \\ ? \end{cases}$
$y = -\frac{5x}{2} + 7$

53. Write a system of linear equations with the solution set $\{(x, y) \mid y = 5x + 2\}$.
53. Answers may vary. Sample:
$\begin{cases} -10x + 2y = 4 \\ 5x - y = -2 \end{cases}$

54. Critical Thinking Look back through the exercises on the previous two pages to find several dependent systems. What relationship exists between the equations in each system? **They are the same equation written in different forms.**

pages 120–123 Exercises

37b. (15, 9); the point represents where the cost of using the bank or online service would be the same.

c. The local bank would be cheaper if you only have 12 bills to pay per month.

Vocabulary Tip

A widget is a small, unspecified gadget.

55. Economics Research shows that in a certain market only 2000 widgets can be sold at \$8 each, but if the price is reduced to \$3, then 10,000 can be sold.

a. Let p represent price and n represent the number of widgets. Identify the independent variable and the dependent variable.

b. Use the information above to write a linear *demand* equation.

c. A shop can make 2000 widgets for \$5 each and 20,000 widgets for \$2 each. Use this information to write a linear *supply* equation.

d. Find the equilibrium point where supply is equal to demand and profit is a maximum. Explain the meaning of the coordinates of this point within the context of the exercise.

a. p: independent, n: dependent
b. $n = -1600p + 14{,}800$
c. $n = -6000p + 32{,}000$
d. About (3.91, 8545); profits are maximized if about 8545 widgets are sold for about \$3.91 each.

Test Prep

Multiple Choice

56. Which is an equation for Line 2? **C**

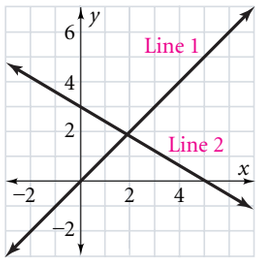
A. $3x - 5y = 15$ **B.** $3x + 5y = 3$
C. $3x + 5y = 15$ **D.** $5x + 3y = 15$

57. Which is NOT an equation for Line 1? **G**

F. $y = x$ **G.** $x + y = 0$
H. $x - y = 0$ **J.** $y - x = 0$

58. Which point lies on both Line 1 and Line 2? **B**

A. (0, 0) **B.** (1.875, 1.875)
C. (1.95, 1.95) **D.** (2, 2)



Exercises 56–58.

59. What is the solution of the system? $\begin{cases} 5x + 6y = -24 \\ -2x + 3y = 15 \end{cases}$ **H**

F. (6, −1) **G.** (6, 1) **H.** (−6, 1) **J.** (−6, −1)

Short Response

60. Explain how you can use slopes to show that the system $\begin{cases} 2x - 5y = 23 \\ 3y - 7x = -8 \end{cases}$ is NOT inconsistent.

Extended Response

61. One equation of a system of equations is $2x - 3y = 5$.

a. Find a second equation such that the system is dependent.

b. Find a second equation such that the system is inconsistent.

60–61. See margin.

Mixed Review

GO for Help

Lesson 2–7 **Graph each inequality on a coordinate plane.** **62–64. See back of book.**

62. $3x - 4y \geq 16$ **63.** $-5x > 8y + 4$ **64.** $x < -4$

Lesson 2–2 **Write an equation for each line.**

65. $m = -\frac{2}{3}$; contains $(-9, 4)$ $y = \frac{-2x}{3} - 2$

66. $m = 0$; contains $(3, 4)$ $y = 4$

67. $m = 2$; contains $(-2, -3)$ $y = 2x + 1$

68. $m = -\frac{1}{2}$; contains $(2, -6)$ $y = -\frac{1}{2}x - 5$

Lesson 1–3 **Solve each equation and check the solution.**

69. $3n = -4(2 + n)$ $-\frac{8}{7}$

70. $-4a + a = 7a - 6$ $\frac{3}{5}$

71. $\frac{x}{3} + 5 = \frac{1}{6}$ -14.5

72. $4x - 2 = \frac{1}{2}x$ $\frac{4}{7}$

73. $\frac{r}{5} + 5 = r - 3$ 10

74. $2(m - 3) = -4$ 1

Test Prep

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies

60. [2] (The slope of $2x - 5y = 23$ is $\frac{2}{5}$ and the slope of $3y - 7x = -8$ is $\frac{7}{3}$. Since the slopes are not equal, the lines are not parallel and they do not coincide. So the lines intersect; the system has exactly one solution and is consistent.
[1] does not include explanation

61. [4] (a) A second equation is $4x - 6y = 10$, or any equation of the form $2ax - 3ay = 5a$.
(b) A second equation is $2x - 3y = 6$ or any equation of the form $2ax - 3ay = 5b$, where $a \neq b$.
[3] minor error in either part (a) or (b)
[2] minor error in both parts (a) and (b)
[1] only completes part (a) or (b)

44a. $\begin{cases} c = 20d + 30 \\ c = 25d \end{cases}$

c. The Pooch Pad would be cheaper for a 7-day stay.

50. An independent system has one solution. The slopes are different, but the y-intercepts could be the same. An inconsistent system has no solution. The slopes are the same, and the y-intercept are different. A dependent system has an infinite number of solutions. The slopes and y-intercepts are the same.

3-2 Solving Systems Algebraically

What You'll Learn
- To solve a system by substitution
- To solve a system by elimination

. . . And Why
To find the cost of joining a health club, as in Example 2

Check Skills You'll Need
GO for Help Lessons 1-1 and 1-3

Find the additive inverse of each term.

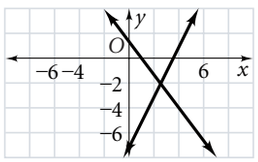
1. 4 -4
2. $-x$ x
3. $5x$ $-5x$
4. $8y$ $-8y$

Substitute $2y - 1$ for x in each equation. Solve for y.

5. $x + 2y = 3$ 1
6. $y - 2x = 8$ -2
7. $2y + 3x = -5$ $-\frac{1}{4}$

New Vocabulary
- equivalent systems

1 Solving Systems by Substitution



$\begin{cases} 4x + 3y = 4 \\ 2x - y = 7 \end{cases}$

Not every system can be solved easily by graphing. Consider the system at the left. Although you can graph each line easily, the exact point of intersection is not obvious. Substitution allows you to find exact solutions without using a graphing calculator.

1 EXAMPLE Solving by Substitution

Solve the system by substitution. $\begin{cases} 4x + 3y = 4 \\ 2x - y = 7 \end{cases}$

Step 1 Solve for one of the variables. Solving the second equation for y is easiest.

$$2x - y = 7$$
$$y = 2x - 7$$

Step 2 Substitute the expression for y into the other equation. Solve for x.

$$4x + 3y = 4$$
$$4x + 3(2x - 7) = 4 \quad \text{Substitute for } y.$$
$$4x + 6x - 21 = 4 \quad \text{Distributive Property}$$
$$4x + 6x = 25$$
$$x = 2.5$$

Step 3 Substitute the value of x into either equation. Solve for y.

$$y = 2x - 7$$
$$y = 2(2.5) - 7 \quad \text{Substitute for } x.$$
$$y = -2$$

The solution is $(2.5, -2)$.

Quick Check
1. Solve each system by substitution. Check your answers.

a. $\begin{cases} 2x - 3y = 6 \\ x + y = -12 \end{cases}$ $(-6, -6)$

b. $\begin{cases} 3x - y = 0 \\ 4x + 3y = 26 \end{cases}$ $(2, 6)$

Differentiated Instruction Solutions for All Learners

Special Needs L1
Many students find the value of one variable and stop, thinking they are done. Draw a pair of intersecting lines and illustrate that the solution is given by the point of intersection, which is an ordered pair (x, y).
learning style: visual

Below Level L2
Have students explain the substitution method orally in their own words. Then have students explain the elimination method orally in their own words.
learning style: verbal

3-2

1. Plan

Objectives
1 To solve a system by substitution
2 To solve a system by elimination

Examples
1 Solving by Substitution
2 Real-World Connection
3 Solving by Elimination
4 Solving an Equivalent System
5 Solving a System Without a Unique Solution

Professional Development

Math Background
Solving systems algebraically is more exact than solving by graphing. The methods of substitution and elimination both result in the solution of an equation in one variable. That solution is then used to find the value of another variable, and so on. The substitution method is most helpful when one equation is presented as solved for one of the variables. The method of elimination is a good way to begin when the coefficients of a variable are opposites.

More Math Background: p. 116C

Lesson Planning and Resources
See p. 116E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice
Check Skills You'll Need
For intervention, direct students to:

Properties of Real Numbers
Lesson 1-1: Example 4
Extra Skills and Word Problems Practice, Ch. 1

Solving Equations
Lesson 1-3: Example 2
Extra Skills and Word Problems Practice, Ch. 1

2. Teach

Guided Instruction

1 EXAMPLE Auditory Learners

Have students explain the substitution method orally in their own words.

Teaching Tip
In Quick Check 1a, students can solve just as easily for x or for y. You might want to show on the board how both methods result in the same answer, $(-6, -6)$.

Error Prevention!

When using substitution, some students correctly solve one equation for a variable, but incorrectly substitute the variable expression into the *same* original equation. Tell students they solve *one* equation (for one variable), and substitute the resulting variable expression into the *other* equation.

PowerPoint

Additional Examples

1 Solve the system by substitution. **(2.1, 3.3)**

$$\begin{cases} x + 3y = 12 \\ -2x + 4y = 9 \end{cases}$$

2 At Renaldi's Pizza, a soda and two slices of the pizza-of-the-day cost \$10.25. A soda and four slices of the pizza-of-the-day cost \$18.75. Find the cost of each item. **soda: \$1.75; pizza: \$4.25**

2 EXAMPLE Real-World Connection

Fees Refer to the advertisement at the left. The cost of membership in a health club includes a monthly charge and a one-time initiation fee. Find the monthly charge and the initiation fee.

Relate $2 \cdot$ monthly charge $+$ initiation fee $= \$100$
$6 \cdot$ monthly charge $+$ initiation fee $= \$200$

Define Let m = the monthly charge. Let f = the initiation fee.

Write $\begin{cases} 2m + f = 100 \\ 6m + f = 200 \end{cases}$

$2m + f = 100$ — Solve for one of the variables.
$f = -2m + 100$
$6m + (-2m + 100) = 200$ — Substitute the expression for f into the other equation. Solve for m.
$m = 25$
$2(25) + f = 100$ — Substitute the value of m into one of the equations. Solve for f.
$f = 50$

- The monthly charge is \$25, and the initiation fee is \$50.

Quick Check 2 **Shopping** You can buy CDs at a local store for \$15.49 each. You can buy them at an online store for \$13.99 each plus \$6 for shipping. Solve a system of equations to find the number of CDs that you can buy for the same amount at the two stores.

$\begin{cases} c = 15.49x \\ c = 6 + 13.99x \end{cases}$ **The number of CDs is 4.**

2 Solving Systems by Elimination

You can solve a system of equations using the Addition Property of Equality. If the quantities you add contain a pair of additive inverses, you can eliminate a variable. You can also eliminate a variable by subtracting like terms.

3 EXAMPLE Solving by Elimination

Test-Taking Tip
You can grid the answer in columns 1–3 or columns 2–4.

Gridded Response Find the y-value in the solution of $\begin{cases} 4x - 2y = 7 \\ x + 2y = 3 \end{cases}$.

$4x - 2y = 7$
$x + 2y = 3$ — Two terms are additive inverses, so add.
$5x = 10$
$x = 2$ — Solve for x.
$x + 2y = 3$ — Choose one of the original equations.
$2 + 2y = 3$ — Substitute for x.
$y = \frac{1}{2}$ — Solve for y.

1	/	2	
	1	1	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

- The solution is $\left(2, \frac{1}{2}\right)$. The y-value is $\frac{1}{2}$.

Quick Check 3 Solve each system by elimination.

a. $\begin{cases} 3x - 2y = 14 \\ 2x + 2y = 6 \end{cases}$ **(4, −1)**

b. $\begin{cases} 4x + 9y = 1 \\ 4x + 6y = -2 \end{cases}$ **(−2, 1)**

Advanced Learners L4
You have \$13.50 to buy one pound of coffee. Blend A costs \$16 a pound. Blend H costs \$8 per pound. How many ounces of each can you buy? **11 oz, 5 oz**

learning style: verbal

English Language Learners ELL
For Example 2, some students may be unfamiliar with health clubs. Make sure students understand what an *initiation fee* is. You may want to ask how the term is related to the word *initial*.

learning style: verbal

To make two terms additive inverses, you may need to multiply one or both equations in a system by a nonzero number. In doing so, you create a system equivalent to the original one. **Equivalent systems** are systems that have the same solution(s).

4 EXAMPLE Solving an Equivalent System

GO Online

Video Tutor Help
Visit: PhSchool.com
Web Code: age-0775

Solve the system below by elimination.

$$\begin{cases} 3x + 7y = 15 \\ 5x + 2y = -4 \end{cases}$$

To eliminate the y terms, make them additive inverses by multiplying.

① $3x + 7y = 15$	$6x + 14y = 30$	Multiply ① by 2.
② $5x + 2y = -4$	$-35x - 14y = 28$	Multiply ② by −7.
	$-29x = 58$	Add.
	$x = -2$	Solve for x.
	$3x + 7y = 15$	Choose an original equation.
	$3(-2) + 7y = 15$	Substitute the value of x.
	$-6 + 7y = 15$	Simplify.
	$7y = 21$	
	$y = 3$	Solve for y.

The solution is $(-2, 3)$.

4. Answers may vary. Sample: Multiply (1) by −5. Multiply (2) by 3. Add equations together, solving for y. Substitute the value of y into either original equation and solve for x.

Quick Check 4 Explain how to solve the system in Example 4 by eliminating x.

Solving a system algebraically does not always result in a unique solution, as in Examples 3 and 4. You may get an equation that is always true, or one that is never true.

5 EXAMPLE Solving a System Without a Unique Solution

Online active math

For: Special Solutions Activity
Use: Interactive Textbook, 3-2

Solve each system by elimination.

a. $\begin{cases} 2x - y = 3 \\ -2x + y = -3 \end{cases}$

$0 = 0$

Elimination gives an equation that is always true. The two equations in the system represent the same line. The system has an infinite number of solutions: $\{(x, y) \mid y = 2x - 3\}$.

b. $\begin{cases} 2x - 3y = 18 \\ -2x + 3y = -6 \end{cases}$

$0 = 12$

Elimination gives an equation that is always false. The two equations in the system represent parallel lines. The system has no solution.

Quick Check 5 Solve each system by substitution or elimination.

a. $\begin{cases} -3x + 5y = 7 \\ 6x - 10y = -14 \end{cases}$ infinite number of solutions $\{(x, y) \mid -3x + 5y = 7\}$

b. $\begin{cases} -2x + 4y = 6 \\ -3x + 6y = 8 \end{cases}$ no solution

Guided Instruction

3 EXAMPLE Math Tip

Make sure students notice that it helps to write both equations in standard form when using the elimination method.

4 EXAMPLE Teaching Tip

Ask students how they would solve the system by eliminating the x terms. **Answers may vary. Sample: Multiply the first equation by 5, and multiply the second equation by −3.**

5 EXAMPLE Alternative Method

If a student suspects the two lines are the same or parallel, have them find the slopes first by solving each equation in a system for y.

PowerPoint

Additional Examples

3 Use the elimination method to solve the system. **(−2, −3)**

$$\begin{cases} 3x + y = -9 \\ -3x - 2y = 12 \end{cases}$$

4 Solve the system by elimination. **(4, −3)**

$$\begin{cases} 2m + 4n = -4 \\ 3m + 5n = -3 \end{cases}$$

5 Solve each system by elimination.

a. $\begin{cases} -3x + 5y = 6 \\ 6x - 10y = 0 \end{cases}$ **no solution**

b. $\begin{cases} -3x + 5y = 6 \\ 6x - 10y = -12 \end{cases}$ $\left\{(x,y) \mid y = \frac{3}{5}x + \frac{6}{5}\right\}$

Resources
- Daily Notetaking Guide 3-2 L3
- Daily Notetaking Guide 3-2—Adapted Instruction L1

Closure

Have students tell how you know whether a system is ready to be solved by elimination. **The terms involving one of the variables, when written on the same side of the equal sign, are additive inverses.**

3. Practice

Assignment Guide

1 A B	1-17, 64-65
2 A B	18-63
C Challenge	63-66
Test Prep	67-72
Mixed Review	73-81

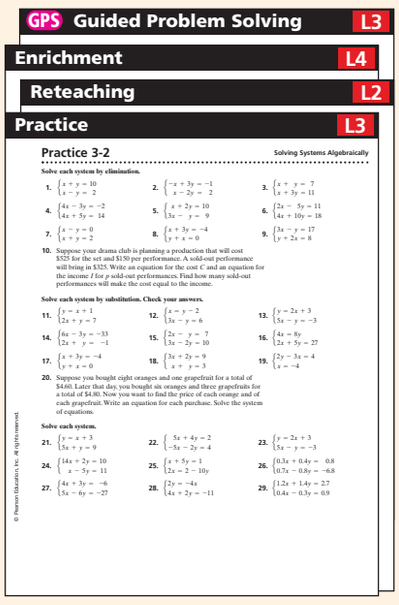
Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 13, 33, 43, 56, 62.

Teaching Tip

Alert students that for systems with variables other than *x* and *y*, the convention is to list the solution in alphabetical order. So, for Exercise 2 the solution is of the form (*c*, *d*).

Differentiated Instruction Resources



EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 125)

GO for Help

Solve each system by substitution. Check your answers.

1. $\begin{cases} 4x + 2y = 7 \\ y = 5x \end{cases}$ (0.5, 2.5)
2. $\begin{cases} 3c + 2d = 2 \\ d = 4 \end{cases}$ (*c*, *d*) = (−2, 4)
3. $\begin{cases} x + 12y = 68 \\ x = 8y - 12 \end{cases}$ (20, 4)
4. $\begin{cases} 4p + 2q = 8 \\ q = 2p + 1 \end{cases}$ (*p*, *q*) = (0.75, 2.5)
5. $\begin{cases} x + 3y = 7 \\ 2x - 4y = 24 \end{cases}$ (10, −1)
6. $\begin{cases} x + 6y = 2 \\ 5x + 4y = 36 \end{cases}$ (8, −1)
7. $\begin{cases} 3a + b = 3 \\ 2a - 5b = -15 \end{cases}$ (*a*, *b*) = (0, 3)
8. $\begin{cases} t = 2r + 3 \\ 5r - 4t = 6 \end{cases}$ (*r*, *t*) = (−6, −9)
9. $\begin{cases} y = 2x - 1 \\ 3x - y = -1 \end{cases}$ (−2, −5)
10. $\begin{cases} 2m + 4n = 10 \\ 3m + 5n = 11 \end{cases}$ (*m*, *n*) = (−3, 4)
11. $\begin{cases} -6 = 3x - 6y \\ 4x = 4 + 5y \end{cases}$ (6, 4)
12. $\begin{cases} r + s = -12 \\ 2r - 3s = 6 \end{cases}$ (*r*, *s*) = (−6, −6)

Example 2 (page 126)

13a. $\begin{cases} d = 0.50m \\ d = 15 \end{cases}$

13. **Fund-Raising** Suppose you have signed up for a bike-a-thon to raise money for charity. One person is sponsoring you at a rate of $.50 per mile. Each of the other sponsors plans to donate $15 no matter how far you bike.
   a. Write a system of equations to model the donation *d* for *m* miles biked.
   b. For how many miles will all sponsors donate the same amount? 30 miles

14. **Transportation** A youth group with 26 members is going skiing. Each of the five chaperones will drive a van or a sedan. The vans can seat seven people, and the sedans can seat five people. How many of each type of vehicle could transport all 31 people to the ski area in one trip? 3 vans and 2 sedans, or 4 vans and 1 sedan, or 5 vans and 0 sedans

15a. $\begin{cases} p = 28 \\ p = 8 + 0.35d \end{cases}$

15. Suppose you have a part-time job delivering packages. Your employer pays you at a flat rate of $7 per hour. You discover that a competitor pays employees $2 per hour plus $.35 per delivery.
   a. Write a system of equations to model the pay *p* for *d* deliveries. Assume a four-hour shift.
   b. How many deliveries would the competitor's employees have to make in four hours to earn the same pay you earn in a four-hour shift? 58

16. A boat can travel 24 mi in 3 h when traveling with a current. Against the same current, it can travel only 16 mi in 4 h. Find the rate of the current and the rate of the boat in still water. 2 mi/h, 6 mi/h

17. **Geometry** The measure of one acute angle of a right triangle is 30° more than twice the measure of the other acute angle. Find the measures of the angles. 20°, 70°, 90°

Example 3 (page 126)

Solve each system by elimination.

18. $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$ (7, 5)
19. $\begin{cases} x + 2y = 10 \\ x + y = 6 \end{cases}$ (2, 4)
20. $\begin{cases} 3a + 4b = 9 \\ -3a - 2b = -3 \end{cases}$ (*a*, *b*) = (−1, 3)
21. $\begin{cases} 4x + 2y = 4 \\ 6x + 2y = 8 \end{cases}$ (2, −2)
22. $\begin{cases} 2w + 5y = -24 \\ 3w - 5y = 14 \end{cases}$ (*w*, *y*) = (−2, −4)
23. $\begin{cases} 3u + 3v = 15 \\ -2u + 3v = -5 \end{cases}$ (*u*, *v*) = (4, 1)
24. $\begin{cases} x + 3y = 11 \\ x + 4y = 14 \end{cases}$ (2, 3)
25. $\begin{cases} 5x + 3y = 30 \\ 3x + 3y = 18 \end{cases}$ (6, 0)
26. $\begin{cases} x - 14 = -y \\ x - y = 2 \end{cases}$ (8, 6)
27. $\begin{cases} 3x + 2y = 6 \\ 3x + 3 = y \end{cases}$ (0, 3)
28. $\begin{cases} 5x - y = 4 \\ 2x - y = 1 \end{cases}$ (1, 1)
29. $\begin{cases} 2r + s = 3 \\ 4r - s = 9 \end{cases}$ (*r*, *s*) = (2, −1)

pages 128–130 Exercises

30. $\{(x, y): -2x + 3y = 13\}$
31. $\{(a, d) \mid -3a + d = -1\}$
32. $(a, b) = (3, 2)$
33. no solution
34. (5, 4)
35. no solution
36. $\left(\frac{20}{17}, \frac{19}{17}\right)$
37. (−3, 2)
38. $(r, s) = (4, 1)$
39. (1, 3)
40. no solution
41. $(m, n) = (1, -4)$

Examples 4, 5 (page 127)

Solve each system by elimination. 30–41. See margin p. 128.

30. $\begin{cases} 4x - 6y = -26 \\ -2x + 3y = 13 \end{cases}$ **31.** $\begin{cases} 9a - 3d = 3 \\ -3a + d = -1 \end{cases}$ **32.** $\begin{cases} 2a + 3b = 12 \\ 5a - b = 13 \end{cases}$

33. $\begin{cases} 2x - 3y = 6 \\ 6x - 9y = 9 \end{cases}$ **34.** $\begin{cases} 20x + 5y = 120 \\ 10x + 7.5y = 80 \end{cases}$ **35.** $\begin{cases} 6x - 2y = 11 \\ -9x + 3y = 16 \end{cases}$

36. $\begin{cases} 2x - 3y = -1 \\ 3x + 4y = 8 \end{cases}$ **37.** $\begin{cases} 5x - 2y = -19 \\ 2x + 3y = 0 \end{cases}$ **38.** $\begin{cases} r + 3s = 7 \\ 2r - s = 7 \end{cases}$

39. $\begin{cases} y = 4 - x \\ 3x + y = 6 \end{cases}$ **40.** $\begin{cases} 3x + 2y = 10 \\ 6x + 4y = 15 \end{cases}$ **41.** $\begin{cases} 3m + 4n = -13 \\ 5m + 6n = -19 \end{cases}$

GO for Help

For a guide to solving Exercise 42, see p. 131.

42. Elections In a mayoral election, the number of votes for the incumbent was 25% more than the number for the opponent. Altogether, the two candidates received 5175 votes. How many votes did the incumbent mayor receive? 2875 votes

43. Writing Explain how you decide whether to use substitution or elimination to solve a system. See margin.

B Apply Your Skills

Solve each system.

44. $\begin{cases} 5x + y = 0 \\ 5x + 2y = 30 \end{cases}$ (−6, 30) **45.** $\begin{cases} 2m = -4n - 4 \\ 3m + 5n = -3 \end{cases}$ **46.** $\begin{cases} 7x + 2y = -8 \\ 8y = 4x \end{cases}$

47. $\begin{cases} v = 9t + 300 \\ v = 7t + 400 \end{cases}$ **48.** $\begin{cases} 80x + 60y = 85 \\ 100x - 40y = 20 \end{cases}$ **49.** $\begin{cases} 2x + 3y = 0 \\ 7x = 3(2y) + 3 \end{cases}$

50. $\begin{cases} \frac{x}{3} + \frac{4y}{3} = 300 \\ 3x - 4y = 300 \end{cases}$ (300, 150) **51.** $\begin{cases} 0.02a - 1.5b = 4 \\ 0.5b - 0.02a = 1.8 \end{cases}$ (a, b) = (−235, −5.8) **52.** $\begin{cases} 4y = 2x \\ 2x + y = \frac{x}{2} + 1 \end{cases}$ (0.5, 0.25)

53. Multiple Choice The equation $3x - 4y = 2$ and which equation below form a system with no solutions? A

(A) $2y = 1.5x - 2$ (B) $2y = 1.5x - 1$

(C) $3x + 4y = 2$ (D) $4y - 3x = -2$

For each system, choose the method of solving that seems easier to use. Explain why you made each choice. 54–59. See margin.

54. $\begin{cases} 3x - 5y = 26 \\ -2x - 3y = -11 \end{cases}$ **55.** $\begin{cases} y = \frac{2}{3}x - 3 \\ -x + 3y = 18 \end{cases}$ **56.** $\begin{cases} 2m + 3n = 12 \\ -5m + n = -13 \end{cases}$

57. $\begin{cases} 3x - y = 5 \\ y = 4x + 2 \end{cases}$ **58.** $\begin{cases} 2x - 3y = 4 \\ 2x - 5y = -6 \end{cases}$ **59.** $\begin{cases} 6x - 3y = 3 \\ 5x - 5y = 10 \end{cases}$

GO Online Homework Help

Visit: PHSchool.com
Web Code: age-0302

60. Open-Ended Write a system of equations in which both equations must be multiplied by a nonzero number before using elimination. Solve your system. See margin p. 130.

61. Internet Access The ads at the left show the costs of Internet access for two companies. a–d. See margin p. 130.

a. Write a system of equations to represent the cost c for t hours of access in one month for each company.

b. Graph the system from part (a). Label each line.

c. For how many hours of use will the costs for the companies be the same? How is this information represented on the graph?

d. If you use the Internet about 20 hours each month, which company should you choose? Explain how you reached an answer.

43. In determining whether to use substitution or elimination to solve an equation, look at the equations to determine if one is solved or can be easily solved for a particular variable. If that is the case, substitution can easily be used. Otherwise, elimination might be easier.

54. Elimination; substitution would be difficult since no coefficient is 1.

55. Substitution; the first equation is solved for y.

4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Solve by substitution. $\begin{cases} -2x + 5y = -2 \\ x - 3y = 3 \end{cases}$ (−9, −4)

2. A bookstore took in $167 on the sale of 5 copies of a new cookbook and 3 copies of a new novel. The next day it took in $89 on the sale of 3 copies of the cookbook and 1 copy of the novel. What was the price of each book? cookbook: $25; novel: $14

Solve each system.

3. $\begin{cases} 10x + 6y = 0 \\ -7x + 2y = 31 \end{cases}$ (−3, 5)

4. $\begin{cases} 7x + 5y = 18 \\ -7x - 9y = 4 \end{cases}$ (6.5, −5.5)

5. $\begin{cases} -3x + y = 6 \\ 6x - 2y = 25 \end{cases}$ no solutions

Alternative Assessment

Ask three students each to state a different equation in standard form. Write the equations where all students can see them. Instruct half of the students to solve the system by finding x first. Instruct the other half to find y first. Have students compare solutions.

56. Substitution; the second equation is easily solved for n.

57. Substitution; the second equation is solved for y.

58. Elimination; $2x$ would be eliminated from the system if the equations were subtracted.

59. Elimination; substitution would be difficult since no coefficient is 1.

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources

For additional practice with a variety of test item formats:

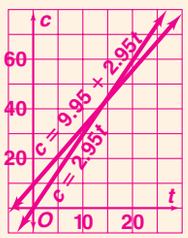
- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies

pages 128–130 Exercises

60. Answers may vary. Sample:
$\begin{cases} -3x + 4y = 12 \\ 5x - 3y = 13 \end{cases}$
(8, 9)

61a. $c = 9.95 + 2.25t$, $c = 2.95t$

b.



c. 14.2 h; It is where the graphs intersect.

d. Answers may vary. Sample: Internet Action, because it would cost $4.05 less per month

76. $y = (x + 3) - 4$ or $y = x - 1$

77. $y = |x - 2| + \frac{1}{2}$

78. $y = 2(x - 1) - 4$ or $y = 2x - 6$

79. $y = |x + 3| + 6$

62. Break-Even Point A theater production costs $40,000 plus $2800 per performance. A sold-out performance brings in $3675. How many sold-out performances will the production need to break even? **46 performances**

Challenge

63. Weather The equation $F = \frac{9}{5}C + 32$ relates temperatures on the Celsius and Fahrenheit scales. Does any temperature have the same number reading on both scales? If so, what is the number? **yes; for −40 degrees**

Find the value of a that makes each system a dependent system.

64. $\begin{cases} y = 3x + a \\ 3x - y = 2 \end{cases}$ **−2**

65. $\begin{cases} 3y = 2x \\ 6y - a - 4x = 0 \end{cases}$ **0**

66. $\begin{cases} y = \frac{x}{2} + 4 \\ 2y - x = a \end{cases}$ **8**

Test Prep

Gridded Response

Use the following system of equations for Exercises 67–70.

$\begin{cases} 5x - 3y = 11 \\ -x + 12y = 3.5 \end{cases}$

67. If you want to solve the system by eliminating x (with addition), by what would you multiply the second equation? **5**

68. If you want to solve the system by eliminating y (with addition), by what would you multiply the first equation? **4**

69. What is the value of x in the solution? Enter your answer as a decimal. **2.5**

70. What is the value of y in the solution? Enter your answer as a decimal. **0.5**

Use the following system of equations for Exercises 71–72.

$\begin{cases} 4x - 10y = -3 \\ 12x + 5y = 12 \end{cases}$

71. What is the value of x in the solution? Enter your answer as a fraction in simplest form. $\frac{3}{4}$

72. What is the value of y in the solution? Enter your answer as a decimal. **0.6**

Mixed Review

GO for Help

Lesson 3-1 **Solve each system of equations by graphing.**

73. $\begin{cases} y = 3x + 4 \\ 2y = 6x - 2 \end{cases}$ **no solution**

74. $\begin{cases} -3y = 9x + 1 \\ 6y = -18x - 2 \end{cases}$ **{(x, y) | −9x − 3y = 1}**

75. $\begin{cases} 4x - y = -5 \\ -8x + 2y = 15 \end{cases}$ **no solution**

Lesson 2-6 **Write an equation for each diagonal translation.** **76–79. See margin.**

76. $y = x$, 4 units down, 3 units left

77. $y = |x|$, $\frac{1}{2}$ unit up, 2 units right

78. $y = 2x - 3$, 1 unit down, 1 unit right

79. $y = |x| + 2$, 4 units up, 2 units left

Lesson 1-1

80. What subset(s) of real numbers contain(s) 6? **natural, whole, integer, rational**

81. The sum of the first and last of four consecutive odd integers is 48. What are the four integers? **21, 23, 25, 27**

3-3 Systems of Inequalities

What You'll Learn
- To solve systems of linear inequalities

. . . And Why
To model college entrance requirements, as in Example 3

Check Skills You'll Need
GO for Help Lessons 1-4, 2-5, and 2-7

Solve each inequality.

1. $5x - 6 > 27$ $x > \frac{33}{5}$
2. $-18 - 5y \geq 52$ $y \leq -14$
3. $-5(4x + 1) < 23$ $x > -\frac{7}{5}$

Graph each inequality. 4–9. See back of book.

4. $y \leq 4x - 1$
5. $3y \geq 6x + 3$
6. $-5y + 2x > -5$
7. $y \leq |x|$
8. $y \geq |x + 3|$
9. $y < |x - 2| + 4$

1 Solving Systems of Inequalities

You can model a situation by writing a system of linear inequalities or a system that combines equations and inequalities.

1 EXAMPLE Solving a System by Using a Table

Classroom Management A science class has 6 computers for 20 students. Students have the option of using a computer program to investigate frog biology or using a computer and their graphing calculators to investigate the properties of heat transfer. Each frog lab must have 3 students in a group. Each heat lab must have 4 students in a group. In how many ways can you set up the lab groups?

Relate number of frog lab groups + number of heat lab groups ≤ 6

$3 \cdot$ number of frog lab groups $+ 4 \cdot$ number of heat lab groups $= 20$

Define Let f = the number of frog lab groups.

Let h = the number of heat lab groups.

Write $f + h \leq 6$

$3f + 4h = 20$

The situation is discrete. The replacement values for f and h must be whole numbers.

To solve the system $\begin{cases} f + h \leq 6 \\ 3f + 4h = 20 \end{cases}$ first make a table of values of f and h that solve the inequality.

f	h
0	6, 5, 4, 3, 2, 1, 0
1	5, 4, 3, 2, 1, 0
2	4, 3, 2, 1, 0
3	3, 2, 1, 0
4	2, 1, 0
5	1, 0
6	0

Differentiated Instruction Solutions for All Learners

Special Needs L1
For Example 2, use masking tape to make a coordinate grid on the floor. Let four students use string to model the two boundary lines. Have other students fill in the overlapping area and stand on the "solid" line to model the solution region.
learning style: tactile

Below Level L2
Have students use colored pencils to lightly shade the graph of each linear inequality. Have them describe how this helps show the region containing the solution of the system.
learning style: visual

3-3

1. Plan

Objectives
1 To solve systems of linear inequalities

Examples
1 Solving a System by Using a Table
2 Solving a System by Graphing
3 Real-World Connection
4 Solving a Linear Absolute Value System

Professional Development

Math Background
The graph of a system of inequalities may or may not include parts of the boundaries as part of the solution.

More Math Background: p. 116C

Lesson Planning and Resources
See p. 116E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice
Check Skills You'll Need
For intervention, direct students to:

Solving Inequalities
Lesson 1-4: Example 1
Extra Skills and Word Problems Practice, Ch. 1

Absolute Value Functions and Graphs
Lesson 2-5: Example 1
Extra Skills and Word Problems Practice, Ch. 2

Two Variable Inequalities
Lesson 2-7: Example 1
Extra Skills and Word Problems Practice, Ch. 2

2. Teach

Guided Instruction

1 EXAMPLE Teaching Tip

Some students may not understand why an inequality is required because no key phrases such as *at most*, or *no more than* are used. Help these students identify the limitation on the number computers available.

2 EXAMPLE

Error Prevention!

Call attention to the graph of $x - 2y < 6$. Point out that students often shade below the boundary line when they see $<$. Note that this only applies when the inequality has the form $y <$, and when solved for y the resulting inequality is $y > -\frac{1}{2}x - 6$.

Teaching Tip

Contrast the types of solutions in Example 1 and Example 2. Ask:

- *How many solutions does Example 1 have?* **two**
- *How many solutions does Example 2 have?* **an infinite number**

3 EXAMPLE Technology Tip

Urge students to be careful with boundary points when interpreting graphs that use the shading feature of the graphing calulator.

4 EXAMPLE

Error Prevention!

Some students may confuse the graphs of linear equations of the form $y < c$ with graphs of $x < c$. Help students recognize that $y < 4$ intersects y at 4, thus having a horizontal boundary line. Similarly, $x < c$ intersects x at c and has a vertical boundary line.

Math Tip

Ask: *What is the solution to a system of two inequalities whose shaded regions do not overlap?* **no solution**

In that table look for values of f and h that solve the equation. Circle any that you find.

The only two whole number solutions of the system are $(0, 5)$ and $(4, 2)$.

There are two possible ways to set up the lab groups. You can assign all students to 5 heat lab groups, or you can assign them to 4 frog lab groups and 2 heat lab groups.

f	h
(0)	6, (5), 4, 3, 2, 1, 0
1	5, 4, 3, 2, 1, 0
2	4, 3, 2, 1, 0
3	3, 2, 1, 0
(4)	(2), 1, 0
5	1, 0
6	0

Quick Check

1. Use tables to solve each system. Assume that replacement values for the variables are whole numbers. **See left.**

a. $\begin{cases} -x + y = 1 \\ x + 2y \leq 20 \end{cases}$ **b.** $\begin{cases} x - y \geq 1 \\ 2x + 3y \leq 21 \end{cases}$

1a. (0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)

1b. (1, 0), (2, 1), (2, 0), (3, 2), (3, 1), (3, 0), (4, 3), (4, 2), (4, 1), (4, 0), (5, 3), (5, 2), (5, 1), (5, 0), (6, 3), (6, 2), (6, 1), (6, 0), (7, 2), (7, 1), (7, 0)

You can solve a system of linear inequalities by graphing. Recall from Lesson 2-7 that when the variables represent real numbers, the solutions of an inequality include all the points on one side of a boundary line. Thus, for two inequalities, every point in the region of overlap of the two solutions is a solution of the system.

2 EXAMPLE Solving a System by Graphing

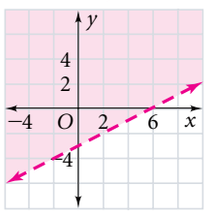
Solve the system of inequalities. $\begin{cases} x - 2y < 6 \\ y \leq -\frac{3}{2}x + 5 \end{cases}$

Problem Solving Hint

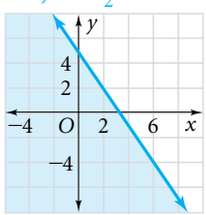
After graphing a boundary line, test whether (0, 0) satisfies the inequality. If it does, shade the side of the line containing (0, 0).

Graph each inequality. First graph the boundary lines. Then decide which side of each boundary line contains solutions and whether the boundary line is included.

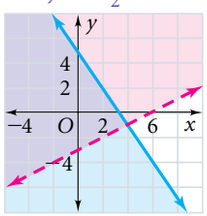
$x - 2y < 6$



$y \leq -\frac{3}{2}x + 5$



$\begin{cases} x - 2y < 6 \\ y \leq -\frac{3}{2}x + 5 \end{cases}$



Every point in the red region above the dashed line is a solution of $x - 2y < 6$.

Every point in the blue region or on the solid line is a solution of $y \leq -\frac{3}{2}x + 5$.

Every point in the purple region where the red and blue regions intersect is a solution of the system. For example, $(1, 1)$ is a solution.

Check Check $(1, 1)$ in both inequalities of the system.

$x - 2y < 6$

$1 - 2(1) < 6$

$-1 < 6$ ✓

$y \leq -\frac{3}{2}x + 5$

$1 \leq -\frac{3}{2}(1) + 5$

$1 \leq \frac{7}{2}$ ✓

Quick Check

2. Solve each system of inequalities. **a–b. See back of book.**

a. $\begin{cases} y \leq -2x + 4 \\ x > -3 \end{cases}$ **b.** $\begin{cases} y \leq 3x - 6 \\ y > -4x + 2 \end{cases}$

Differentiated Instruction Solutions for All Learners

Advanced Learners L4

Have students create a real–world example to be solved with a linear system. Let them exchange exercises and solve.

learning style: verbal

English Language Learners ELL

Many students have difficulty translating a word problem into a system of equations and/or inequalities. Have students work in pairs or small groups and sketch illustrations to represent the situation described in each real-world problem.

learning style: verbal

3 EXAMPLE Real-World Connection

Real-World Connection

Most college admission requirements include standardized test scores.

College Admissions An entrance exam has two parts, a verbal part and a mathematics part. You can score a maximum total of 1600 points. For admission, the school of your choice requires a math score of at least 600. Write a system of inequalities to model scores that meet the school's requirements. Then solve the system.

Relate verbal score + math score ≤ 1600

math score ≥ 600

Define Let x = the verbal score.

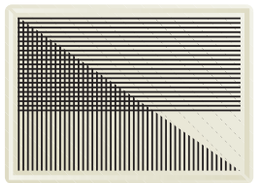
Let y = the mathematics score.

Write $x + y \leq 1600$, or $y \leq 1600 - x$

$y \geq 600$

The system of inequalities is $\begin{cases} y \leq 1600 - x \\ y \geq 600 \end{cases}$.

Xmin = 0 Ymin = 10
Xmax = 1600 Ymax = 1600



Use a graphing calculator to graph the corresponding equations $y = 1600 - x$ and $y = 600$. Since the first inequality is $\leq$, shade below the first line. Since the second inequality is $\geq$, shade above the second line. The region of overlap is a graph of the solution.

Quick Check

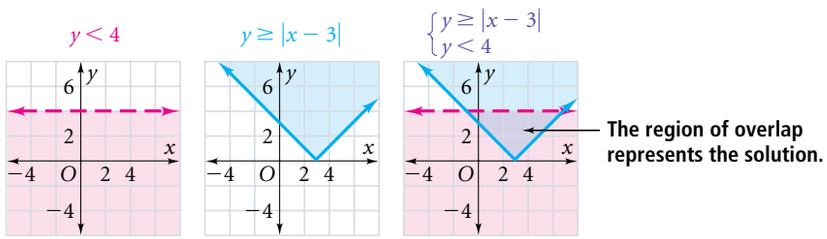
3 Another school requires a math score of at least 550 points and a total score of at least 1100 points. You can score up to 800 points on each part. Write and solve a system of inequalities to model scores that meet the school's requirements. **See back of book.**

Some systems consist of linear and absolute value inequalities.

4 EXAMPLE Solving a Linear Absolute Value System

For: Inequality Systems Activity
Use: Interactive Textbook, 3-3

Solve the system of inequalities. $\begin{cases} y < 4 \\ y \geq |x - 3| \end{cases}$



Quick Check

4 Solve each system of inequalities. **a–b. See back of book.**

a. $\begin{cases} y \geq x \\ y \leq |x + 5| - 2 \end{cases}$ **b.** $\begin{cases} y \geq -2x + 4 \\ y \leq |x - 4| \end{cases}$

c. Critical Thinking Write a system that includes an absolute value inequality but has no solutions. **Check students' work.**

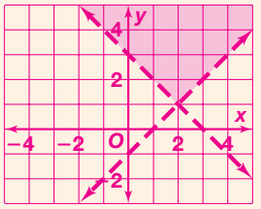
2. Teach

PowerPoint

Additional Examples

1 Find whole number solutions of the system using tables.
$\begin{cases} x + y = 2 \\ 2x + y \leq 5 \end{cases}$ **(0, 2), (1, 1), (2, 0)**

2 Solve the system of inequalities.
$\begin{cases} -x + y > -1 \\ x + y > 3 \end{cases}$



3 Jenna spends at most 150 min a night on math and science homework. She spends at least 60 min on math. Write and solve a system of inequalities to model how she allots her time for these two subjects. **See back of book.**

4 Solve the system.
$\begin{cases} y \geq 3 \\ y > -|x + 2| + 5 \end{cases}$
See back of book.

Resources
- Daily Notetaking Guide 3-3 L3
- Daily Notetaking Guide 3-3—Adapted Instruction L1

Closure

Ask: *How do you solve a system of inequalities by graphing?* **Graph the individual inequalities. Shade the region where the graphs overlap.**

3. Practice

Assignment Guide

1 A B	1-48
C Challenge	49-52
Test Prep	53-56
Mixed Review	57-72

Homework Quick Check

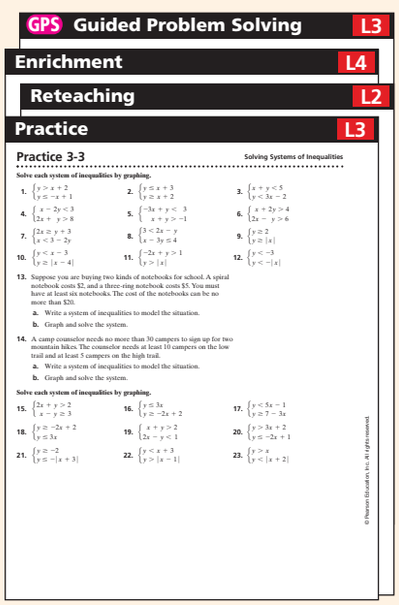
To check students' understanding of key skills and concepts, go over Exercises 10, 35-40, 45.

Error Prevention!

Exercises 12, 13, 15 Students should be careful to have *y,* not $-y$, on the left side of each inequality when determining whether to shade above or below a boundary line.

Exercises 43-45 Students' diagrams should make it clear that the solutions of the systems are the points whose coordinates satisfy *all three* inequalities.

Differentiated Instruction Resources



EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 133)

GO for Help

Example 2 (page 134)

1. (0, 4), (0, 5), (0, 6), (0, 7), (0, 8)

2. (3, 0), (4, 1), (5, 2), (6, 3), (7, 4)

Find the whole number solutions of each system using tables.

1. $\begin{cases} y + 3x \le 8 \\ y - 3 > 2x \end{cases}$ 1–2. See left.

2. $\begin{cases} x = y + 3 \\ x + y \le 12 \end{cases}$

3. $\begin{cases} x + y < 8 \\ 3x \le y + 6 \end{cases}$ See back of book.

Tell whether $(-3, 3)$ is a solution of each system.

4. $\begin{cases} y \ge x + 2 \\ 3y < -6x + 6 \end{cases}$ yes

5. $\begin{cases} y - 2x \le 1 \\ y < -2x - 2 \end{cases}$ no

6. $\begin{cases} -2y + x \le 4 \\ 3y < -9x + 3 \end{cases}$ yes

Solve each system of inequalities by graphing. 7–9. See margin.

7. $\begin{cases} y \le 2x + 2 \\ y < -x + 1 \end{cases}$

8. $\begin{cases} y > -2 \\ x < 1 \end{cases}$

9. $\begin{cases} y \le 3 \\ y \le \frac{1}{2}x + 1 \end{cases}$

10. $\begin{cases} y \le 3x + 1 \\ -6x + 2y > 5 \end{cases}$

11. $\begin{cases} x + 2y \le 10 \\ x + y \le 3 \end{cases}$

12. $\begin{cases} -x - y \le 2 \\ y - 2x > 1 \end{cases}$

13. $\begin{cases} y > -2x \\ 2x - y \ge 2 \end{cases}$

14. $\begin{cases} c \ge d - 3 \\ c < \frac{1}{2}d + 3 \end{cases}$

15. $\begin{cases} 2x + y < 1 \\ -y + 3x < 1 \end{cases}$

10–17. See back of book.

Example 3 (page 135)

16. **Fund-Raising** You want to bake at least 6 and at most 11 loaves of bread for a bake sale. You want at least twice as many loaves of banana bread as nut bread. Write and graph a system of inequalities to model the situation.

17. **Psychology** A psychologist needs at least 40 subjects for her experiment. She cannot use more than 30 children. Write and graph a system of inequalities.

Example 4 (page 135)

Solve each system of inequalities by graphing. 18–29. See back of book.

18. $\begin{cases} y > 4 \\ y < |x - 1| \end{cases}$

19. $\begin{cases} y < -\frac{1}{3}x + 1 \\ y > |2x - 1| \end{cases}$

20. $\begin{cases} y > x - 2 \\ y \ge |x + 2| \end{cases}$

21. $\begin{cases} y \le -\frac{4}{3}x \\ y \ge -|x| \end{cases}$

22. $\begin{cases} 3y < -x - 1 \\ y \le |x + 1| \end{cases}$

23. $\begin{cases} y > -2 \\ y \le -|x - 3| \end{cases}$

24. $\begin{cases} -2x + y > 3 \\ y \le -|x + 4| \end{cases}$

25. $\begin{cases} 5y \ge 2x - 5 \\ y < |x + 3| \end{cases}$

26. $\begin{cases} y \ge -3x + 3 \\ y > |x + 2| \end{cases}$

27. $\begin{cases} -2y < 4x + 2 \\ y > |2x + 1| \end{cases}$

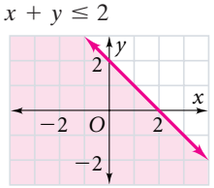
28. $\begin{cases} -x \ge 4 - y \\ y \ge |3x - 6| \end{cases}$

29. $\begin{cases} y \le x - 4 \\ y > |x - 6| \end{cases}$

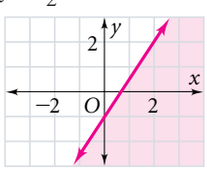
B Apply Your Skills

In Exercises 30–39, identify the inequalities A, B, and C for which the given ordered pair is a solution.

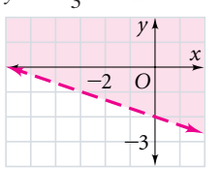
A. $x + y \le 2$



B. $y \le \frac{3}{2}x - 1$



C. $y > -\frac{1}{3}x - 2$

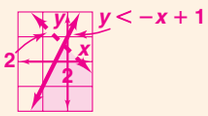


GO Online Homework Help
Visit: PHSchool.com
Web Code: age-0303

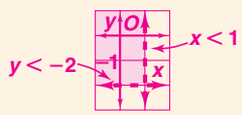
30. $(0, 0)$ A, C
31. $(-2, -5)$ A, B
32. $(-2, 0)$ A, C
33. $(0, -2)$ A, B
34. $(-15, 15)$ A, C
35. $(3, 2)$ B, C
36. $(2, 0)$ A, B, C
37. $(-6, 0)$ A
38. $(4, -1)$ B, C
39. $(-8, -11)$ A

pages 136–138 Exercises

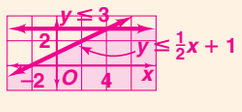
7. $y \le 2x + 2$; $y < -x + 1$



8. $y < -2$; $x < 1$



9. $y \le 3$; $y \le \frac{1}{2}x + 1$



Real-World Connection

Bake sales are a popular way to raise money.

40. Fund-Raising GPS Suppose the Student Council has asked you to form a committee to run a bake sale. The committee needs from 7 to 10 members. The number of seniors should be greater than the number of juniors.

a. Write a system of inequalities to model the problem.

b. Graph the system and list the combinations of juniors and seniors that may participate in the committee. **a–c. See back of book.**

c. Critical Thinking Explain why your list in part (b) is finite.

41. Open-Ended Write and graph a system of inequalities for which the solution is bounded by a dashed vertical line and a solid horizontal line.

42. Writing Explain how you determine where to shade when solving a system of inequalities.

41–51. See back of book.

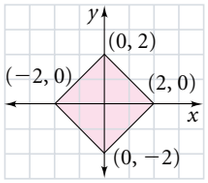
Solve each system of inequalities by graphing.

43. $\begin{cases} x + y < 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$ **44.** $\begin{cases} 2y - 4x \leq 0 \\ x \geq 0 \\ y \geq 0 \end{cases}$ **45.** $\begin{cases} y \geq -2x + 4 \\ x > -3 \\ y \geq 1 \end{cases}$

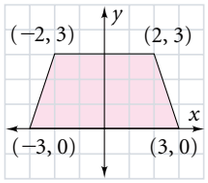
46. $\begin{cases} y \leq \frac{2}{3}x + 2 \\ y \geq |x| + 2 \end{cases}$ **47.** $\begin{cases} y < x - 1 \\ y > -|x - 2| + 1 \end{cases}$ **48.** $\begin{cases} 2x + y \leq 3 \\ y > |x + 3| - 2 \end{cases}$

Challenge **Geometry** **Write a system of inequalities to describe each shaded figure.**

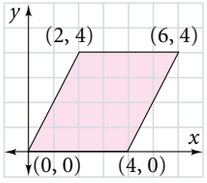
49.



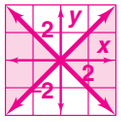
50.



51.



52a.



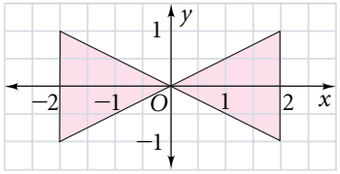
52. a. Graph the "bowtie" inequality, $|y| \leq |x|$.

b. Write a system of inequalities to describe the graph shown at the right.

Answers may vary.

Sample: $|y| \leq \frac{1}{2}|x|$

$|x| \leq 2$



Test Prep

Multiple Choice

53. When you graph Inequality ① at the right, the boundary line should be ? and the shading should be ? the line. **B**

① $\begin{cases} y < -2x + 3 \\ y \geq x - 4 \end{cases}$

A. dashed, above **B.** dashed, below **C.** solid, above **D.** solid, below

54. When you graph Inequality ②, the boundary line should be ? and the shading should be ? the line. **H**

F. dashed, above **G.** dashed, below **H.** solid, above **J.** solid, below

55. What is the x-value of the intersection of the boundary lines? **D**

A. $\frac{-7}{3}$ **B.** $\frac{-3}{7}$ **C.** $\frac{3}{7}$ **D.** $\frac{7}{3}$

Short Response

56. How would you test whether (2, −2) is a solution of the system?

See back of book.

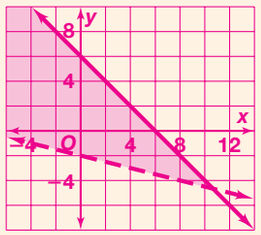
4. Assess & Reteach

PowerPoint

Lesson Quiz

1. Solve the system of inequalities by graphing.

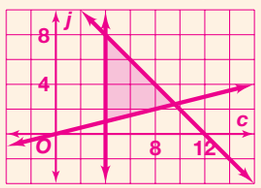
$\begin{cases} x + y \leq 6 \\ -x - 4y < 8 \end{cases}$



2. A 24-hour radio station plays only classical music, jazz, talk programs, and news. It plays at most 12 h of music per day, of which at least 4 h is classical. Jazz gets at least 25% as much time as classical. Write and graph a system of inequalities.

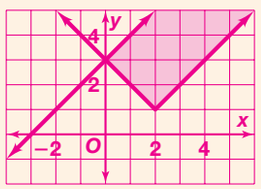
Let c = hours for classical and j = hours for jazz.

$c + j \leq 12$, $c \geq 4$, $j \geq 0.25c$



3. Solve the system of inequalities by graphing.

$\begin{cases} y \leq x + 3 \\ y \geq |x - 2| + 1 \end{cases}$



Alternative Assessment

Have students work in pairs. Each student writes one inequality for a system. They combine the two inequalities to make a system, and each graphs the system. They check one another's work.

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 165
- Test-Taking Strategies, p. 160
- Test-Taking Strategies with Transparencies

Checkpoint Quiz

Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 3-1 through 3-3

Resources
Grab & Go
- Checkpoint Quiz 1

Mixed Review

GO for Help

Lesson 3-2 **Solve each system by elimination or substitution.**

57. $\begin{cases} y = 3x + 1 \\ 2x - y = 8 \end{cases}$ $(-9, -26)$ **58.** $\begin{cases} 3x + y = 4 \\ 2x - 4y = 7 \end{cases}$ $\left(\frac{23}{14}, -\frac{13}{14}\right)$ **59.** $\begin{cases} -x + 5y = 3 \\ 2x - 10y = 4 \end{cases}$ no solution

60. $\begin{cases} 2x + 4y = -8 \\ -5x + 4y = 6 \end{cases}$ $(-2, -1)$ **61.** $\begin{cases} y - 3 = x \\ 4x + y = -2 \end{cases}$ $(-1, 2)$ **62.** $\begin{cases} 2 = 4y - 3x \\ 5x = 2y - 3 \end{cases}$ $\left(-\frac{4}{7}, \frac{1}{14}\right)$

Lesson 2-3 **For each function, y varies directly as x.**

63. If $y = -6$ when $x = -2$, find y when $x = 3$. 9

64. If $y = -8$ when $x = 2$, find x when $y = 2$. $-\frac{1}{2}$

65. If $y = 4$ when $x = 7$, find y when $x = -14$. -8

66. If $y = 9$ when $x = 15$, find x when $y = 6$. 10

Lesson 1-5 **Solve each equation. Check your answers.**

67. $|2x + 5| = 6$ $\frac{1}{2}, -\frac{11}{2}$ **68.** $|x + 7| = -2$ no solution

69. $3|x - 4| + 1 = 13$ 8, 0 **70.** $-2|x + 1| - 5 = -7$ $-2, 0$

71. $\frac{1}{2}|3x + 2| - 3 = 4$ $-\frac{16}{3}, 4$ **72.** $-|2x + 5| = -3$ $-4, -1$

Checkpoint Quiz 1 — Lessons 3-1 through 3-3

Solve each system of equations.

1. $\begin{cases} 3x + 2y = 6 \\ x - 2y = 10 \end{cases}$ $(4, -3)$ **2.** $\begin{cases} 4x + 7y = 28 \\ y = 2x - 14 \end{cases}$ $(7, 0)$

3. $\begin{cases} 4x + 5y = -12 \\ 3x - 4y = 22 \end{cases}$ $(2, -4)$ **4.** $\begin{cases} 3y - 2x = 7 \\ 2y - 2 = 4x \end{cases}$ $(1, 3)$

5. The Village Inn offers two special packages. For two nights and three meals the cost is \$158. For two nights and five meals the cost is \$181. Write and solve a system of linear equations to find the costs per night and per meal.

5. $\begin{cases} 2n + 3m = 158 \\ 2n + 5m = 181 \end{cases}$ \$61.75 cost per night, \$11.50 cost per meal

6. **Smart Shopping** An ordinary refrigerator costs \$489 and has an estimated annual operating cost of \$84. An energy-saving model costs \$599, with an estimated annual cost of \$61. After how many years will the costs to buy and to operate the two models be equal? approximately 4.8 years

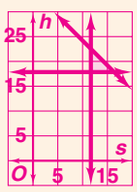
7. Each week you must do a minimum of 18 hours of homework. Participation in sports requires at least 12 hours per week. You have no more than 35 hours per week in total to devote to these activities. a–b. See margin.
a. Write a system of inequalities to model the situation.
b. Graph and solve the system.

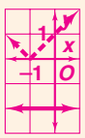
Solve each system of inequalities by graphing. 8–10. See margin.

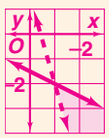
8. $\begin{cases} y \le -2 \\ y > |x + 1| \end{cases}$ **9.** $\begin{cases} 8x + 2y > 5 \\ x + 2y \le -3 \end{cases}$ **10.** $\begin{cases} 4y < 3x - 1 \\ y > 2|x| - 3 \end{cases}$

page 138 Checkpoint Quiz 1

7a. $\begin{cases} h \ge 18 \\ s \ge 12 \\ s + h \le 35 \end{cases}$

b. 

8. 

9. 

10.